Geometry—the study of shapes, their properties, and the spaces containing them—has always been one of the most important branches of mathematics. Today, computer graphics, computational techniques, and a wide range of applications are giving new importance to geometry. In addition, there are new developments in geometry every year, ranging from abstract theoretical results to the discovery of the geometric structure of soap bubbles, the buckyball, and quasicrystals. Yet, for most students, even those who take further mathematics courses in college, high school geometry is all the geometry that they will every formally study.

The purpose of this paper is to describe the ways students will use the geometry they have learned in high school once they have graduated. High school geometry provides an important and necessary basis for a wide range of college majors and professions, and we will see that the geometry recommendations of the NCTM Standards serve a wide range of students well. Since geometry and visualization skills are beneficial and useful for all students, they should all get strong preparation in geometry.

**Geometry in the College Curriculum**

It is rare for a college student to take a course in geometry and at many schools it is possible to major in mathematics without taking even one course in geometry. Nevertheless, students in almost every major will use geometry in one way or another. We begin by looking at the geometry required in a number of standard mathematics courses and then we will consider other disciplines and how they use geometry, geometric reasoning, and visualization skills.

**Calculus.** Calculus is the study of quantities that change continuously with respect to one another. Traditionally, it is the first mathematics course many college freshmen take. Calculus has earned its reputation as one of the most difficult classes primarily because of the new concepts introduced, but calculus also demands proficient algebra skills and a strong geometry background.
Analytic geometry is used throughout calculus, to go back and forth between visual, geometric representations of functions, their secants, tangents, and other related constructions on the one hand and the algebraic representations of these on the other hand. Geometry often provides greater intuitive understanding while algebra provides computational techniques. To understand the key concepts of continuity and limits, students must be familiar with the geometric structure of the real number line. Students can be convinced that points do not lie “next to” each other on the number line is familiar if they know how to bisect a line segment with compass and straightedge. The property that there are no gaps or holes on the number line is related to the idea that two lines intersect at a point.

The concept of derivative requires an understanding of the tangent line not as a line that intersects a circle at one point, as it presented in geometry. More generally, a tangent line is the line that best approximates a curve at a given point or looks more like the curve at that point than any other line. Similarly, a secant is a line that intersects any curve at two or more points, not just a line that intersects a circle at two points. These ideas can be introduced by looking at tangents and secants to the other shapes: the conic sections or the graphs of higher-degree polynomials.

The definition of derivative in terms of limits depends on knowing how to find the slope of a line given two points on the line. These points have variables as coordinates, so students should know how to work with points whose coordinates are variables rather than specific numbers.

Knowledge of trigonometry is also important for calculus. To obtain the derivatives of the trigonometric functions, students will use the definitions of the trigonometric functions in terms of the unit circle. All of the common trigonometric identities are eventually used in calculus: in obtaining the derivatives of tangent and secant from the derivatives of sine and cosine, obtaining the derivatives of the inverse trigonometric functions, integrating trigonometric functions, or in applications.

The usual applications of derivatives in optimization and related rates problems require students to be familiar with area and volume formulas. Often, the shapes involved in these problems must be broken into simpler shapes such as semicircles, rectangles, triangles, hemispheres, and cones.

In the definition of the integral, the area under a curve is broken down into small trapezoids. Applications of integration to finding volumes, arc lengths, and surface areas use formulas for
area, volume, and surface area of many simple geometric shapes, including the circle, annulus, cone, prism, pyramid, and cylinder. Solids of revolution appear frequently in volume and surface area problems. Students must visualize these solids and then decide whether to use shells or slices to compute their volume or surface area. The ability to understand and visualize three-dimensional solids is very important here and many students have difficulty with this.

The Pythagorean theorem, as one would expect, appears in many places in a calculus course. It is often used in optimization and related rates problems and is essential to understanding the integrals representing arc length and surface area.

In more advanced calculus courses, students will use vectors and three-dimensional analytic geometry. These topics are usually presented as if students have not had much exposure to them and are taught as needed. However, students will be at an advantage if they have had prior exposure to vectors and graphing in three dimensions, either in mathematics or physics courses.

Statistics. Many students take statistics in college and even here, geometry and visualization skills come into play. When working with probability distributions, students will be computing the area of a region as the sum or difference of areas of simpler regions. They must also be able to estimate the location of a line that bisects an area, for example, that under a skewed distribution.

Linear Algebra. A course in linear algebra is usually required for mathematics, computer science, physics, and engineering majors. Many topics in geometry—analytic geometry, parallel lines and perpendicular lines, transformations and their representations by matrices, vectors, orientation, and three-dimensional visualization—provide a good foundation for the study of linear algebra.

Linear Programming. In business, economics, and operations research, linear programming is used to optimize a quantity, for example to maximize profit or minimize cost, subject to constraints on resources. A feasible region of acceptable possibilities is determined as the intersection of the solution sets of the linear inequalities that represent constraints. A feasible region is thus a polygon in two-dimensional space or a polytope in higher-dimensional space. The object function, representing a quantity like profit or cost, defines a hyperplane in the same space. To understand linear programming, students must know how to graph lines and inequalities in the plane and they should have had some experience with three-dimensional analytic geometry and hyperplanes.
**Geometry.** A strong high school preparation in geometry is essential for students who will study geometry in college. Such a preparation should include not only standard material in Euclidean geometry, but also an introduction to other types of geometries, whether non-Euclidean geometry, projective geometry, topology, or even taxicab metrics. Geometry encompasses an imposing array of material (see Gorini, 2003) and students should get a taste of this in high school. Most college geometry courses include a variety of different geometries; some even begin with a geometry other than Euclidean to emphasize the fact that Euclidean geometry is but one of many geometries.

**Abstract Algebra.** Even the study of abstract algebra at the college level can benefit from a good geometry background. Many abstract algebra books start with a group as the set of symmetries of a geometrical shape or design. Prior understanding of symmetry transformations is helpful, and the ability to identify rotation, reflection, translations, glide reflections, and their compositions can assist in overcoming the abstraction of group theory.

These mathematics courses all make essential use of geometry and are taken by students in many different majors. In addition, courses in other disciplines use geometrical reasoning and visualization skills. We will now look at a number of different college majors and what they require of students in terms of geometry.

**Mathematics.** It goes without saying that a student desiring to study mathematics at the college level should learn as much geometry as possible at the high school level. Since it is sometimes possible to major in mathematics without taking a course in geometry, prospective mathematics majors should take as much geometry at the high school level as possible, including non-Euclidean geometries.

**Engineering.** Engineering programs are challenging and demand a strong mathematical background from students. Engineering students will use geometry in many of their courses: mathematics, particularly calculus and linear algebra, physics, and drawing and design.

**Sciences.** Most science majors require calculus, so potential science majors should have the geometry preparation recommended for calculus. They will be using analytic geometry in data analysis and many science students take courses in statistics. The sciences all require good
visualization skills, particularly for working with three-dimensional shapes. In chemistry and physics, students must determine the appropriate coordinate system—rectangular, polar, cylindrical or spherical—and its origin based on the symmetry of a particular shape in a problem. In chemistry students will work with symmetries of three-dimensional molecules. Physics demands the strongest mathematical preparation of all the sciences and requires knowledge of Euclidean geometry, analytic geometry, and solid geometry.

**Commerce, Business, and Economics.** Programs in these areas vary widely, but all require a good knowledge of analytic geometry, particularly lines and their slopes. Many of these programs also require courses in statistics, calculus, or linear programming.

**Visual Arts.** There are many avenues in the visual arts: fine art, architecture, graphic design, multimedia, and computer graphics. Programs and courses in the visual arts do not generally have mathematical prerequisites or requirements, so students in these programs will be relying completely on their high school geometry training. For such students, all areas of geometry are valuable. They will use standard Euclidean geometry for recognizing and understanding shapes; geometric techniques in perspective drawing; symmetry for use in design; proportion and similarity for drawing; and three-dimensional geometry for sculpture.

**Geometry in Careers**

There are many ways that students will use geometry and visual thinking in their future careers, whether or not they go to college. In many occupations, people already use computer graphics and computational geometry, and will use even more in the future. The few examples given below indicate the extensive range of applied geometry in use today. For more examples, see Gorini, 2000.

- Engineers design imaging equipment that can convert measurements into pictures. A radiologist interprets the pictures—CAT scans or MRIs—made by this equipment to diagnose patients.
- An engineer programs a robot so that it can navigate in a two- or three-dimensional environment, performing its job and avoiding collisions. The owners of the robot arrange their workroom to accommodate the robot.
• An architect designs a building using CAD software. A structural engineer analyzes the structural components of the building to ensure that it will be strong enough. Construction workers read blueprints drawn by the architect as they construct the building.
• City planners and traffic engineers determine how people, vehicles, and goods can move through the streets and highways of a region without congestion. Civil engineers use data from geographical information systems to plan the best locations for public services and utilities.
• Design engineers use CAD/CAM software to design everything from screws to computer circuits and automobiles.
• A home decorator makes measurements, estimates fabric yardage, plans the placement of designs on drapes or a slipcover, cuts the required pieces, and sews them together.
• Meteorologists interpret geometric models of the weather to make their predictions.
• Mathematics teachers help all kinds of students understand all areas of geometry to prepare for their careers.

**Summary of Important Geometry Knowledge and Skills**

For their study and occupations beyond high school, all students need the basic material that is taught in standard high school geometry courses. They need to recognize different shapes, find areas and volumes, and draw diagrams. They will likely be using the Pythagorean theorem, similar triangles, and analytic geometry. Students intending to go to college should have more geometry, including transformations, three-dimensional geometry, and spherical geometry.

All students should develop geometric reasoning, visualization, and problem solving skills. A proof-based geometry course can help students develop these skills, but there are other approaches that are successful as well. Teachers should choose what is most appropriate for their students.

Teaching visualization skills can be challenging even for experienced teachers. The whole subject of visualization skills, what they are, how they can be taught, and how they can be assessed is very new (Whiteley, 2002). Some kinds of visualization skills that are important in geometry are illustrated in the following list.

- Look at a line in the coordinate plane and estimate its slope: positive or negative, large or small
- Recognize the symmetries of a shape or design
- Find a path through a maze
• Judge which of two shapes has the larger perimeter or area
• Break an area or volume into the sum or difference of simpler areas or volumes
• Draw the intersection of a plane with a solid
• Make a two-dimensional representation of a three-dimensional object
• Build a three-dimensional object from two-dimensional data
• Draw a diagram from a verbal description
• Give a verbal description of a geometric shape
• Create, manipulate, and interpret computer graphics

Experience with these sorts of activities will help students develop visualization skills and be able to apply their geometrical knowledge to new situations.

Conclusion

In almost every college major or career, students will see some geometry and will use visualization skills. The sciences, engineering, and art naturally require more geometry, but even in commerce and business, geometry is used.

It is important for students to have a strong background in geometry and this can be achieved with courses that meet the NCTM Standards (NCTM, 2000) in geometry. As a college teacher, I often hear comments like “I always liked geometry” or “I never liked mathematics except for geometry” from students and my perception is that high school teachers are doing a good job with geometry. If all students can have a year of geometry, or its equivalent, in high school, they will be well prepared for future studies and whatever careers they wish to pursue.

But as important as teaching specific topics is teaching about the spirit of geometry. Students should have fun studying symmetrical patterns and making their own designs. Maybe they can draw some pictures using perspective or they can visit a science museum and talk to each other in a whispering gallery. They will enjoy playing with fractals on a computer, making their own using Sketchpad or Cabri. They can hear about the discovery of buckyballs and quasicrystals. They should work on and solve challenging problems in geometry. They should know that people like geometry, use it, and are still making discoveries in it. They should see the excitement and joy of geometry, they should feel good about their ability to do geometry, and they should be inspired to continue their exploration of geometry.
References


