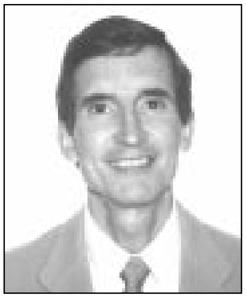
About the Author



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Maharishi's Vedic Mathematics in Elementary Education: Developing All Knowingness to Improve Affect, Achievement, and Mental Computation

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Abstract

Maharishi's Vedic Mathematics is the absolutely precise organizing power inherent in the structure of self-referral consciousness, pure knowledge, the Veda. In elementary education, the computational aspect of Maharishi's Vedic Mathematics is Vedic sūtra based computation, derived from 16 Vedic Sūtras *or aphorisms. Each* sūtra *serves as the central point from which many different algorithms emerge.*

In the context of a full Maharishi's Vedic Mathematics program, Vedic Sūtra based computation cultures the ability of the mind to function from the level of pure, self-referral consciousness—all knowingness, the Unified Field of all the Laws of Nature—while the student computes. This culturing process gradually developes the ability to think and act in accord with all the Laws of Nature. The student begins to compute more in accord with the principle of least action which makes computation easier, faster, more mental, and more enjoyable. Through this developmental process, it is predicted that over time a full Maharishi's Vedic Mathematics program will create the instantaneous solution to any problem, the state of all knowingness—"mathematics without steps"—in the life of a student.

Results from an empirical study comparing Vedic Sūtra based multiplication and checking to conventional methods at the third grade level indicate that students using the Vedic Sūtra based approach have higher achievement scores, retain more multiplication and checking skill, and enjoy computation more than students using conventional methods. The Vedic Mathematics group also computed more efficiently and performed more mental computation. Structured interviews conducted after all students involved in the study had learned both approaches to computation indicated that Vedic Sūtra based computation was easier, more enjoyable, and more motivating than conventional methods.

Contents

Introduction

Maharishi Mahesh Yogi, founder of Maharishi Universities of Management and Maharishi Schools of the Age of Enlightenment, explains that the purpose of education is to "develop wholeness in the life of the learner" (Maharishi Mahesh Yogi, 1995a, p. 145). Ideal education should

> strengthen and integrate the physical, mental, and behavioral aspects of life by developing their common basis, pure consciousness. This creates wholeness of awareness, the expression of a truly integrated life, in which all the material, intellectual, and spiritual aspects of life are beautifully correlated. (Maharishi Mahesh Yogi, 1995a, pp. 145–146)

In Maharishi Vedic Science and Technology℠, pure consciousness refers to the unbounded field of pure intelligence—the universal basis of one's personality and the source of all thought and action (Maharishi Mahesh Yogi, 1995a, 1969). Most important to mathematics education, Maharishi explains that pure consciousness is the Unified Field of all the Laws of Nature, the most fundamental level of Nature's functioning, which governs all phenomena in the universe with absolute order and precision (Maharishi Mahesh Yogi, 1995b). Maharishi concludes that without the intellectual understanding and direct personal experience of pure consciousness, the students' knowledge of Natural Law remains incomplete, without basis. Furthermore, students are unable to gain the practical use of the infinite organizing power of Natural Law in daily life.

Because modern day education throughout the world does not offer a systematic program for research in consciousness and generally is unaware of consciousness and its applied value, teachers have been unable to provide knowledge of the basis of Natural Law. As a result, Maharishi concludes that "education is baseless everywhere, and baseless education can only produce disintegrated, stressful, and frustrated individuals, and a society full of problems and suffering" (Maharishi Mahesh Yogi, 1995a, p. 15). Experienced educators support this perspective. Bloom states that most professors are specialists unable to form the student as a whole or develop the students' "real potential" (Bloom, 1987, p. 339). Nidich and Nidich write that today's schools primarily emphasize the objective knowledge of the academic disciplines "without providing any systematic means to develop the subjective side of the educational process" (Nidich & Nidich, 1990, p. 3). They go on to say that developing the students' full inner potential is left more or less to chance, with the assumption that they will somehow become mature, well-rounded human beings from exposure to the standard curriculum. Furthermore, without the systematic understanding and experience of consciousness, the basis of the personality, much learning becomes meaningless and irrelevant to students' lives. Lack of relevancy leads to a lack of motivation to learn, which in turn produces a substandard level of academic performance and problems with social behavior. Parents find that students are simply not growing, not making use of their full creative potential. Mathematics education unfortunately follows this pattern.

Certainly mathematics education appreciates the centrality of students and their subjective character. This has been demonstrated by its attempts to understand and apply theories of learning and cognitive development (e.g., Kroll, 1989; Hiebert, 1984; Starkey & Gelman, 1982); its desire to understand the role of affective factors such as self-confidence, anxiety, and willingness (e.g., McLeod & Adams, 1989; Reyes, 1984); and its development of theories of metacognition—the knowledge of how the mind thinks from moment to moment and how one gains control over the thought processes (e.g., Pirie & Kieren, 1989; Kilpatrick, 1985; Garofalo & Lester, 1985). However, none of these strategies has succeeded in providing knowledge of the full range of consciousness or how to develop the student's full potential. Due to education's lack of knowledge about consciousness and how to develop it fully, problems persist in mathematics education, a fact consistently born out in discipline assessments.

The 1990 National Assessment of Educational Progress (NAEP) states that "only slightly more than half" of eighth and twelfth graders like mathematics and that enjoyment of mathematics declines with years in school (Mullis, Dossey, Owen, & Phillips, 1991, p. 40). This same finding was noted in the 1986 NAEP which found that student perceptions of the vitality of the subject had not improved from 1978 to 1986 (Dossey, Mullis, Lindquist, & Chambers, 1988, p. 105). Also, a national evaluation of students stated that "less than 20 percent reach proficient levels of mathematics achievement in grades 4, 8, and 12" (National Assessment Governing Board, 1991, p. 1). For example, one-third of the eighth graders tested were unable to use the basic skill of addition to find the cost of three items on a menu (Mullis, Dossey, Owen, & Phillips, 1991, pp. 9–10).

Maharishi's Vedic Mathematics fulfills the need in mathematics education for complete knowledge of consciousness by providing the understanding, experience, and development of consciousness as the student learns mathematics (Maharishi Mahesh Yogi, 1988; Brooks & Brooks, 1988; Indian Institute of Maharishi Vedic Science and Technology, March 1988, fax communication to Maharishi University of Management; Maharishi Mahesh Yogi, 1988, private communication with M. Weinless, Ph.D.; B. Morris, 1988, fax communication to Maharishi University of Management). To understand the value of consciousness to mathematics education, three topics will be examined in separate sections: (1) the Veda, pure knowledge which is self-existent within consciousness and composed of all the Laws of Nature that structure the dynamic evolution of the universe; (2) Maharishi's technologies for the development of consciousness that make it possible to easily access the Veda, and (3) the state of all knowingness, the ability to know anything spontaneously and solve any problem effortlessly without making mistakes, developed by the regular experience of pure consciousness through Maharishi's Vedic Technologies.

Based on the terms and concepts introduced in these three sections, a fourth section introduces the value of Vedic Sūtra based computation. It presents the main thesis of this paper: the use of the Vedic Sūtras (aphorisms), in the context of a full Maharishi's Vedic Mathematics program,[1] cultures the ability to think and act in accord with all the Laws of Nature as the student computes. As the ability to think in harmony with Natural Law

[1]A full Maharishi's Vedic Mathematics program at the elementary level includes the Transcendental Meditation technique, Vedic Sūtra based computation, and may include Maharishi's Absolute Number. Maharishi points out that his Absolute Number "explains the supreme level of reality—the world of wholeness or many wholenesses"; it is "a number that will help us to account for the theme of creation and evolution in terms of wholeness" (Maharishi Mahesh Yogi, 1995b, p. 611). The competence or authority of Maharishi's Absolute Number to fully express the "infinite number of wholenesses within the universe," is due to it being (1) "a meaningful living reality" which may be experienced by any student "in the self-referral wholeness of silence and dynamism—Saṁhitā of Ṛishi, Devatā, and Chhandas," and (2) capable of expressing the togetherness and self-referral relationship of silence and dynamism, unity and diversity, the Self and the universe (Maharishi Mahesh Yogi, 1995b, pp. 611, 620, 625, 630). Undefined terms mentioned above, such as Saṁhitā, are discussed in this article. For a more complete discussion of the Absolute Number and its relationship to modern mathematics, see Maharishi Mahesh Yogi, 1995b, pp. 379–382, 611–643.

grows, I predict that computation will become faster, easier, more enjoyable, more motivating, and more self-referral. For computation to become more self-referral means that it occurs more within the Self, pure consciousness. Initially, mental computation occurs more frequently. Long term, solutions occur spontaneously on the level of consciousness. The fifth and final section of this paper will report a research study that evaluates this thesis among third grade students at Maharishi School of the Age of Enlightenment (Maharishi School) in Fairfield, Iowa.

Maharishi's Description of the Veda

The following discussion presents Maharishi's explanation of how the verses of the Veda are the self-interacting dynamics of consciousness, the Laws of Nature, structured within the consciousness of everyone. It will provide the theoretical basis for understanding how Vedic Sūtra based computation is the link which connects computation to pure consciousness, the most orderly and powerful level of organization in man and Nature.

Self-Interacting Dynamics of Consciousness

In Maharishi's teaching, "consciousness is that which is conscious of itself" (Maharishi Mahesh Yogi, 1995a, p. 53). Being conscious of itself, it is *self-referral*. Self-referral consciousness is the unbounded field of wakefulness, intelligence, all knowingness, complete openness in all directions (Maharishi Mahesh Yogi, 1995a, p. 17). This infinite, self-referral field is generally referred to as pure consciousness or pure intelligence.

Maharishi makes a clear distinction between self-referral consciousness and object-referral consciousness. Object-referral consciousness occurs when consciousness is conscious of something else, such as thoughts, sensory stimuli, or objects in the environment. However, even when consciousness is object-referral, the experience of being conscious of a particular stimuli takes place within self-referral consciousness:

> When we say total reality of consciousness, we mean consciousness in its self-referral state, where consciousness knows itself and nothing else. This state of consciousness is pure consciousness. Another state of consciousness is when it knows other things; then it is known to be object-referral consciousness, because all objects can only be perceived by virtue of the intelligence quality of consciousness, which creates the observer and process of observation within the singularity of the self-referral state of consciousness. This establishes that the object-referral state of consciousness is also within the self-referral state of consciousness. (Maharishi Mahesh Yogi, 1995a, pp. 54–55)

In self-referral consciousness, consciousness knows itself thereby becoming its own knower and known. Since cognition occurs within its own unbounded continuum, consciousness is also its own process of knowing. Thus, in its self-referral state, consciousness is the unified state of knower, knowing, and known. Since consciousness appears as three (knower, knowing, known) while remaining singular (consciousness), it has a "three-in-one" structure (Maharishi Mahesh Yogi, 1995a, pp. 59, 308–310). In the Vedic Language, this three-in-one structure is called Saṁhitā (unity) of Ṛishi (the knower or wakeful aspect of consciousness which cognizes), Devatā (the process of knowing or dynamism inherent in the process of cognition), and Chhandas (the known which hides the knower and process of knowing). The singularity of Saṁhitā is transformed into the diversity of Ṛishi, Devatā, and Chhandas; yet, because Ṛishi, Devatā, and Chhandas are just different modes of self-referral consciousness, the singularity is preserved (Maharishi Mahesh Yogi, 1995a, pp. 59–62). So it is clear that, in the state of self-referral consciousness, singularity is transformed into diversity and diversity into singularity and these transformations occur simultaneously.

The transformation of singularity into diversity and diversity into singularity, Maharishi points out, creates a reverberation or "continuous sound" within the nature of absolute silence, *Atyanta-abhāva* (Maharishi Mahesh Yogi, 1995a, p. 63; 1995d, pp. 165, 423). In Maharishi Vedic Science℠, these reverberations are identified as the Veda, "the sound of self-reverberating consciousness," the "self-generated" sound of consciousness knowing itself. These reverberations in pure consciousness are not isolated. The whole Veda exists simultaneously at every point throughout the unbounded continuum of consciousness.

Maharishi goes on to explain that Vedic Sound spontaneously evolves in a precise, predictable sequence (Maharishi Mahesh Yogi, 1995d, pp. 165–166). As consciousness comes to know itself fully, the perfectly ordered sequence of its self-interacting dynamics gives rise to the "sequentially developing, properly structured, specific sounds developing in sequence within the unified field of consciousness." As the sequence unfolds, first Ṛk Veda[2] sounds are created, then Sāma Veda sounds, Yajur-Veda sounds, Atharva Veda sounds, and then the sounds of all the corresponding Vedic Literature (Maharishi Mahesh Yogi, 1987; Nader, 1995).

The sequence of Vedic Sound *(Shruti,* or "that which is heard") was originally cognized by the ancient Vedic Seers in its pure, undifferentiated state and eventually recorded as all the different aspects of the Vedic Literature (Maharishi Mahesh Yogi, 1995a, p. 63). Thus, Maharishi explains, the various aspects of Vedic Literature emerge from the self-interacting dynamics of Ṛishi, Devatā, and Chhandas within the unity of Saṁhitā. Together, the Vedic Literature displays total knowledge of the dynamic relationships among Saṁhitā and Ṛishi, Devatā, and Chhandas. Thus, Veda is the self-interacting dynamics of consciousness naturally existing at all times within pure consciousness, "intimately personal to everyone" (Maharishi Mahesh Yogi, 1995a, p. 82).

Laws of Nature

Maharishi explains that Vedic Sound, within the absolute silence of consciousness, creates the entire manifest universe including the human nervous system. This process is based on the relationship of sound and form. Each sound has a corresponding form and, furthermore, the sound structures the form (Maharishi Mahesh Yogi, 1995a, pp. 316–317; Nader, 1995; Gelderloos & Berg, 1989, p. 383).

We have seen that Vedic Sound *(Shruti)* exists within consciousness. Due to the relationship of sound and form, stated above, the Vedic Form (*Darshana*) inherent within Vedic Sound also exists within the unified, self-referral structure of pure consciousness. Maharishi points out that "the sound is heard by self-referral consciousness and the form is seen by self-referral consciousness" (Maharishi Mahesh Yogi, 1995a, pp. 317–318).

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[2]In Maharishi Vedic Science, Ṛk Veda is the totality of knowledge about the self-interacting dynamics of consciousness (Maharishi Mahesh Yogi, 1995a, pp. 80–84). Sāma Veda, Yajur-Veda, and Atharva Veda comprise the specific aspects of Natural Law that are engaged in promoting the qualities of Ṛishi, Devatā, and Chhandas, respectively, within Saṁhitā.

Through its self-interacting dynamics, consciousness is sequentially transformed into more diverse sounds and forms (Maharishi Mahesh Yogi, 1995a, pp. 311–320; see also Hagelin, 1989. pp. 38–39). The mechanics of transformation within the structure of Saṁhitā quantifies the holistic nature of Vedic Sound creating the entire universe of alphabets, speech, and books. Maharishi explains further:

> The momentum of transformation that constitutes the process of evolution continues to take the evolution of sound to create, first, the form of the sound itself, the script. The momentum of evolution continues, and what comes out of sound is form. The basic expression of form is a particle of matter. (1995a, pp. 312–313)

Sequentially evolving structures of particles generate congregations of particles. This momentum of evolution continues giving rise to more concrete expressions and the whole manifest, ever expanding universe:

> Consciousness continues to generate qualities one after the other. Saṁhitā generates Ṛishi, Devatā, Chhandas, and all the permutations and combinations of the three in sequential progression of the process of evolution of pure intelligence giving rise to the five senses of perception [hearing, touch, sight, taste, and smell], the five elements [space, air, fire, water, and earth], and from these the whole manifest creation within the nature of self-referral consciousness, promoted and sustained by the self-referral dynamism of consciousness. (Maharishi Mahesh Yogi, 1995a, p. 318)

Being the reverberations which give rise to the entire universe of subjective and objective phenomena, Maharishi identifies the Vedic Verses as "the Laws of Nature" (Maharishi Mahesh Yogi, 1995c, pp. 34–60; 1969, p. 206). This is expressed in Ṛk Veda (1.164.39), *Ṛicho Ak-kshare parame vyoman yasmin devā adhi vishwe nisheduḥ*, which Maharishi translates as, "the verses of the Veda exist in the collapse of fullness in the transcendental field, self-referral consciousness, the Self, in which reside all the *Devās*, the impulses of Creative Intelligence, the Laws of Nature responsible for the whole manifest universe" (B. Morris, private communication, July 11, 1991).

Thus, in Maharishi Vedic Science, the physical universe is just a more precipitated expression of Veda, the self-interacting dynamics of pure consciousness (Maharishi Mahesh Yogi, 1987; 1969, p. 206; Maharishi International University, 1990a, p. 20). This description corresponds with that given by modern quantum physics, in which the particles that structure creation are understood to be excited modes or reverberations of underlying quantum fields (Hagelin, 1989, pp. 20–21).

This discussion has indicated how the Vedic Verses are the Laws of Nature that create the universe; Veda may also be described as pure knowledge, the complete knowledge of Natural Law.

Pure Knowledge and Infinite Organizing Power

Maharishi explains that "knowledge blossoms when the knower comes in contact with the known; or Knowledge blossoms when the subject comes in contact with the object" (Maharishi Mahesh Yogi, 1995a, p. 204). In a separate discussion, Maharishi described the knowledge of a rose to make this point concrete (Maharishi Mahesh Yogi, 1980b, pp. 73–76). The knower's attention falls on the rose and allows the image of the rose to occupy the awareness. At the point where the image of the rose occupies consciousness, consciousness assumes a specific color or flavor which Maharishi identifies as the knowledge of the rose. It is through this unity of the image of the rose (the known) and consciousness (the knower) by means of various mental processes (the process of knowing) that knowledge of the rose is born. The knowledge that blossoms in the union of knower and known pertains to the rose but it exists on the level of consciousness. It is structured in consciousness. Thus, through the integration of knower and known, Maharishi points out that consciousness assumes different roles of perception and is transformed into different structures of knowledge.

Following this line of reasoning, Maharishi explains how the unity of knower, known, and process of knowing in self-referral consciousness gives rise to the Veda, the most fundamental structure of knowledge (Maharishi Mahesh Yogi, 1995a, pp. 73–80; 1980b, p. 74). In its self-referral state, consciousness is the knower of itself; the knower cognizes its own nature; pure consciousness alone occupies the awareness. Since, in this self-referral state, the knower, known, and process of knowing are aspects of consciousness, the knowledge that blossoms resides completely within the field of consciousness. It is structured in consciousness and pertains entirely to consciousness, the home of all the Laws of Nature. Maharishi refers to this self-referral structure of knowledge as the Veda, or "pure knowledge," the complete knowledge of Natural Law (Maharishi Mahesh Yogi, 1995a, pp. 3–10).[3]

According to information theory, greater information about a system reduces its entropy and increases the knowledge of that system (Singh, 1966, pp. 76–77). From this, Maharishi concludes that knowledge increases orderliness: it has organizing power (Maharishi Mahesh Yogi, 1995a, p. 118). The complete knowledge of Natural Law, being infinite, has total or infinite organizing power. Thus, Veda is pure knowledge (*Mantra*), which has infinite organizing power (*Brāhmaṇa*).

Maharishi explains that this holistic structure of pure knowledge, Veda, encompasses all knowledge about creation—its source, course, and goal—the whole range of science and technology. He points out that Vedic Knowledge

> includes the whole path of knowledge from the knower to the known—the whole field of subjectivity, objectivity, and their relationship; the whole field of life, unmanifest and manifest; the whole field of "Being" and "Becoming"; the whole range of knowledge from its source to its goal—the eternal source, course, and goal of all knowledge. (1995a, p. 5)

Once the mind becomes fully open to the Veda in pure consciousness, Maharishi explains, the student would possess all knowledge: "If the full value of pure intelligence, the unbounded, infinite value of consciousness, could become an all-time reality in our conscious mind, then our mind would be the home of all knowledge" (Maharishi Mahesh Yogi, 1972, p. 9:5). When the student's mind is the home of all knowledge, Maharishi states, one gains "the fruit of all knowledge," the ability to know anything spontaneously and achieve anything without making mistakes. Not surprisingly, then, students who regularly gain access to the Veda through the experience of pure consciousness begin to improve academically without increased effort (see research cited below). Maharishi points out that this experience of consciousness is the highest and most complete form of education:

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[3]Pure knowledge and pure consciousness are two terms that identify for the same unbounded, self-referral field experienced at the source of thought, Saṁhitā of Ṛishi, Devatā, and Chhandas (Maharishi Mahesh Yogi, 1995a,).

> Development of consciousness, or gaining supreme level of consciousness, or gaining self-referral consciousness, is complete education, which is the state of all knowingness, the ability to spontaneously know anything, do anything right, and achieve anything at will. This ability to achieve anything at will in the state of self-referral consciousness, singularity, is the ability to spontaneously engage infinite Creative Intelligence, Cosmic Creative Intelligence, to bring fulfillment to all desires. (1995a, p. 112)

Infinite Creative Intelligence refers to the unlimited creative potential of the Veda that gives rise to the entire universe through its own self-interacting dynamics. Gaining complete knowledge, the state of all knowingness, through the full development of consciousness is not only the highest aim of education, it is the supreme state of fulfillment for every individual. Achievement of all knowingness is gained through Maharishi's Vedic Technologies discussed in the following sections.

Development of Consciousness through Maharishi's Vedic Technologies

Maharishi's Vedic Mathematics includes the Transcendental Meditation® (TM®) and TM-Sidhi® programs which systematically cultures the experience of pure consciousness in the student. The experience of pure consciousness gives access to pure knowledge and its infinite organizing power which, in turn, maximizes the effectiveness of Vedic Sūtra based computation.

Experience of Pure Consciousness

> Maharishi's Transcendental Meditation technique is a simple, natural, effortless procedure practiced for 15 to 20 minutes in the morning and evening while sitting comfortably with eyes closed. During this technique the individual's awareness settles down and experiences a unique state of restful alertness: as the body becomes deeply relaxed, the mind transcends all mental activity to experience the simplest form of human awareness—Transcendental Consciousness—where consciousness is open to itself. This is the self-referral state of consciousness. (Maharishi Mahesh Yogi, 1995a, pp. 260–261)

In the Vedic Language, Transcendental Consciousness is referred to as *Ātmā*, the universal Self—the basis of ego, emotions, intellect, mind, and senses (Maharishi Mahesh Yogi, 1995a; 1972, p. 19:3–12; see also 1986, p. 299; 1969, pp. 118–138, 421–422; 1963, p. 44; Roth, 1987).

The experience of pure consciousness provided by the Transcendental Meditation technique has been described physiologically as a unique state of restful alertness (Maharishi Mahesh Yogi, 1995c, pp. 174–185; Wallace, 1993, 1986, 1970). The body becomes deeply rested while the mind remains fully alert. Deep physiological rest can be objectively verified. One way is by measuring the decreased respiratory rate during the Transcendental Meditation technique and periods of respiratory suspension during the experience of Transcendental Consciousness (Dillbeck & Orme-Johnson, 1987; Badawi, Wallace, Orme-Johnson, & Rouzere, 1984; Wolkove, Kreisman, Darragh, Cohen, & Frank, 1984; Farrow & Herbert, 1982). Measurements of alpha and theta EEG power and coherence during practice of the Transcendental Meditation technique indicates alertness and/or orderliness (Farrow & Herbert, 1982; Dillbeck & Bronson, 1981; Orme-Johnson & Haynes, 1981; Banquet & Sailhan, 1974; Orme-Johnson, Wallace, Dillbeck, Alexander, & Ball, 1981). Anecdotally, personal testimonies of the experience of completely settled states of awareness during the Transcendental Meditation technique indicates that pure consciousness is a field of bliss (*ānanda*) or infinite happiness (International Association for the Advancement of the Science of Creative Intelligence, 1976, pp. 74–85; Maharishi Mahesh Yogi, 1963, p. 22).

The TM-Sidhi program is an advanced technology based on the experience of pure consciousness established by the Transcendental Meditation technique. The TM-Sidhi program enlivens pure consciousness and develops the habit of thinking and acting from the level of pure consciousness (Maharishi Mahesh Yogi, 1995a, pp. 261–288). Scientific research noted below has found that the TM-Sidhi program produces greater coherence in brain functioning—a condition associated with higher levels of creativity, intelligence, flexibility of mind, neurological efficiency, and moral reasoning.

Children under 10 years of age practice the Maharishi Word of Wisdom℠ technique. This technique is designed to strengthen the nervous system, give students the experience of more orderly and blissful aspects of their nature, and to help them focus more effectively in class (Nidich & Nidich, 1990). Scientific research indicates that practice of the Word of Wisdom technique develops field independence, conceptual maturity, sustained attention (Dixon, 1989), as well as information processing ability (Warner, 1986).

Over 500 scientific research studies (Orme-Johnson & Farrow, 1977; Chalmers, Clements, Schenkluhn, & Weinless, 1989; Wallace, Orme-Johnson, & Dillbeck, 1990) conducted at more than 200 universities and research institutions have verified that the Transcendental Meditation and TM-Sidhi programs systematically develop full human potential (e.g., Orme-Johnson, 1988; Alexander & Boyer, 1989). For example, research shows that through the experience of pure consciousness, mental potential increases. One gains:

- increased self-actualization (Nidich, Seeman, & Dreskin, 1973; Alexander, Rainforth, & Gelderloos, 1991);
- increased intelligence among college students (Cranson, Orme-Johnson, Gackenbach, Dillbeck, Jones, & Alexander, 1991; Jones, 1989; Aron, Orme-Johnson, & Brubaker, 1981; Dillbeck, Assimakis, Raimondi, Orme-Johnson, & Rowe, 1986) as well as among primary and secondary students (Nidich & Nidich, 1990, p. 94; Shecter, 1978; Warner, 1986);
- improved memory, learning, and cognitive flexibility (Dillbeck, 1982; Miskiman, 1973; Alexander, Langer, Newman, Chandler, & Davies, 1989);
- increased field independence (Dillbeck, Assimakis, Raimondi, Orme-Johnson, & Rowe, 1986; Pelletier, 1974);
- increased efficiency of information transfer in the brain (Wandhofer, Kobal, & Plattig, 1976; Kobal, Wandhofer, & Plattig, 1975); and
- increased creativity (Travis, 1979; Dillbeck, Landrith, & Orme-Johnson, 1981; Orme-Johnson, 1982).

This growth of mental potential produces more precise, effective thinking, which improves a wide range of variables important to mathematics education. With regular practice of the Transcendental Meditation technique and the TM-Sidhi program, the brain begins to function in a more orderly, coherent manner even during dynamic activity (Dillbeck & Bronson, 1981; Farrow & Herbert, 1982; Gaylord, Orme-Johnson, Willbanks,

& Travis, 1989). Due to the increased coherence, students practicing the Transcendental Meditation technique solve problems more spontaneously (Nidich, Nidich, & Rainforth, 1986; Dillbeck, 1982; Miskiman, 1973) and gain increased ability to think and act efficiently (Frew, 1974; Hjelle, 1974). As their mental potential increases, their health also improves (e.g., Wallace, 1993). Maharishi calls this holistic growth the development of higher states of consciousness. In the highest state, Unity Consciousness, all types of problem solving become perfectly effective, spontaneous, effortless, self-referral (on the level of consciousness), accurate, and blissful.

Higher States of Consciousness

Maharishi has delineated seven states of consciousness which constitute the full range of human development (Maharishi Mahesh Yogi, 1995a, p. 161). These states are deep sleep consciousness or *Sushupti Chetanā*, dreaming consciousness or *Swapn Chetanā*, waking consciousness or *Jāgrat Chetanā*, Transcendental Consciousness or *Turīya Chetanā*, Cosmic Consciousness or *Turīyātīt Chetanā*, God Consciousness or *Bhagavad Chetanā*, and Unity Consciousness or *Brāhmī Chetanā*. The first three states are identified by Maharishi as relative states of consciousness because they are always changing (Maharishi Mahesh Yogi, 1972, pp. 22:6–7; 1986, p. 289). The last four states of consciousness he identifies as "higher states" of consciousness because they are based on the experience of pure consciousness, the "higher," universal Self, which is absolute and unchanging in nature.

In the same way that scholars have evaluated experiences during the Transcendental Meditation technique, research on the higher states of consciousness has been reviewed (Alexander & Boyer, 1989), the implications of higher states of consciousness to the field of developmental psychology have been discussed (Alexander, Davies, Dixon, Dillbeck, Druker, Oetzel, Muehlman, & Orme-Johnson, 1990), and the physiological parameters of the states of consciousness have been presented (Wallace, Fagan, & Pasco, 1988; Wallace, 1993, 1986).

Deep sleep is considered by Maharishi to be the least developed state of consciousness; it is a state of inertia where there is no knowledge of self or environment:

> There are many phases of consciousness: consciousness of the wakeful state; consciousness of the dreaming state, a completely different consciousness; consciousness of the deep sleep state—complete ignorance, full ignorance, to the extent that there is no experience of objectivity and no experience of illusory objectivity [the dream state]. (Maharishi Mahesh Yogi, 1986, p. 288)

In the dreaming state, Maharishi points out that knowledge is unreliable. Perception is illusory. In waking consciousness, the awareness of self and environment is limited by the consciousness of the perceiver; therefore, knowledge and fulfillment are also limited.

Maharishi explains that in the waking state, the knower or self is localized in time and space and separate from the known or objects of experience (Maharishi Mahesh Yogi, 1986; see Alexander & Boyer, 1989, pp. 329–330). This disintegration of knower and known leads to the fragmented and incoherent style of brain functioning known as waking consciousness (Hagelin, 1989, p. 39). That is, waking electroencephalograph (EEG) patterns measured across topologically distinct regions of the brain are significantly less synchronous or coherent than those measured during higher states of consciousness

(Alexander & Boyer, 1989, pp. 338–339). Due to the lack of integration with its source in absolute bliss, the waking mind tends to search continually for satisfaction among the objects of experience (Maharishi Mahesh Yogi, 1969, p. 121). This search for fulfillment is satisfied in the experience of higher states of consciousness.

The first higher state is Transcendental Consciousness. This state of consciousness is directly experienced through the Transcendental Meditation technique. It has been described previously as pure consciousness, self-referral consciousness, Self, Saṁhitā, or *Ātmā*. The experience of Transcendental Consciousness stands as the first step to the development of Unity Consciousness, the goal of all human evolution (Maharishi Mahesh Yogi, 1969, p. 144). The progressive integration of Transcendental Consciousness with activity begins with the development of Cosmic Consciousness, progresses further in God Consciousness, and finds its final fulfillment in Unity Consciousness.

Maharishi calls the fifth state of consciousness Cosmic Consciousness (Maharishi Mahesh Yogi, 1972, p. 23–5). This state is characterized by the spontaneous coexistence of pure consciousness at all times, along with the waking, dreaming, and deep sleep states of consciousness. Maharishi (1986) explains that in this state everything is perfectly harmonious and consciousness is brilliant, clear, full of satisfaction, and blissful. Due to the development of full mental potential, the individual established in Cosmic Consciousness gains "skill in action": goals are attained quickly, with the least effort, and without adverse effects on the environment (Maharishi Mahesh Yogi, 1969, pp. 142–143).

In Cosmic Consciousness, even though one is established in unbounded awareness, Maharishi points out that still one perceives only the surface value of the object (Maharishi Mahesh Yogi, 1972, pp. 23:6–7). The only qualities perceived are those which distinguish the object from the rest of its environment. However, because unbounded awareness is established on the level of the conscious mind, one's perception naturally begins to appreciate deeper and deeper values of the object, until perception is so refined that the finest level of existence is capable of being spontaneously perceived on the gross, surface level throughout waking, dreaming, and sleeping states of consciousness. This ability gives rise to the sixth state of consciousness, God Consciousness, characterized by unbounded awareness along with perception of the finest level of existence. Maharishi refers to the sixth state of consciousness as God Consciousness because one is able to perceive and appreciate the entire range of creation as well as the mechanics of creation seen at the junction point between creation and pure consciousness (Maharishi Mahesh Yogi, 1986, p. 428; see Alexander & Boyer, 1989, p. 355).

In the transition from God Consciousness to Unity Consciousness, the object of perception becomes increasingly appreciated in terms of pure consciousness, the Self. This transition culminates when perception gains its infinite value and every aspect of the object is appreciated in terms of the Self (Maharishi Mahesh Yogi, 1972, p. 23:9; 1969, pp. 441–443). Maharishi explains how this development eliminates the "gulf," or perceived distance, between the knower and known making it possible to know the object of perception completely:

> In this unified state of consciousness, the experiencer and the object of experience have both been brought to the same level of infinite value, and this encompasses the entire phenomenon of perception and action as well. The gulf between the knower and the object of his knowing has been bridged. When the unbounded perceiver is able to cognize the object in its

> total reality, cognizing the infinite value of the object which was hitherto unseen, then the perception can be called total, of supreme value. In this state, the full value of knowledge has been gained, and we can finally speak of complete knowledge. (1972, p. 23:9)

The gulf or separation between the knower and known experienced in Cosmic Consciousness is narrowed in God Consciousness and finally eliminated in Unity Consciousness. Maharishi affirms that there is still a knower and known in the structure of knowledge, but there is a lack of distance between them; the distance is completely unmanifest. Life is lived in unbroken wholeness.

In Unity Consciousness, the complete range of the object of perception is spontaneously and effortlessly available to the knower. The object has a surface level, numerous intermediate levels—such as chemical, nuclear, and subatomic—and pure consciousness at its basis. With the full integration of knower and known in this seventh state of consciousness, the knower is able to spontaneously gain access to that level of the object which will yield the desired knowledge. Being able to know anything spontaneously, the knower gains complete fulfillment. Maharishi elaborates:

> When the awareness of the knower has all these different values open to it, the knower will be able to pick up whatever value of the object is most useful at a given time and place. He will therefore spontaneously be able to make the maximum use of his environment. This ability of spontaneously picking up what is best for this time, in this place, under these circumstances, is a very great gift. (1972, p. 32:8)

In this quote, Maharishi emphasizes spontaneity and practicality. Because the entire range of the object is open to the knower, complete knowledge is gained easily. Furthermore, knowledge in Unity Consciousness is described as flawless. This state of consciousness is defined by Maharishi as the highest goal of human development (Maharishi Mahesh Yogi, 1972, p. 23:10).

Maharishi also describes the growth within Unity Consciousness (Maharishi Mahesh Yogi, 1973; see Alexander & Boyer, 1989, p. 360). When Unity Consciousness first dawns, Maharishi explains, only the primary object of perception is experienced in terms of the Self. Gradually, this structure of experience grows to include the secondary objects of perception, then the tertiary objects, until finally all possible objects of perception are experienced in terms of the Self. Maharishi refers to this fully ripened state of Unity Consciousness as Brahman Consciousness, the supreme wholeness of life, the state of all knowingness. Now let's examine the practical value of all knowingness to mathematics.

Nature of All Knowingness in Maharishi Vedic Science

The development of consciousness has many practical outcomes vital to mathematics education including more orderly, coherent brain functioning and increased ability to think and act efficiently as discussed previously. With the full development of consciousness, Maharishi teaches that students acquire an infinitely powerful administrative skill called "mathematics without steps," or all knowingness (Maharishi Mahesh Yogi, 1995b, pp. 388–389). Due to the unique intellectual character of all knowingness, two prerequisite concepts—infinite correlation and least action—are introduced before beginning the discussion of mathematics without steps.

Infinite Correlation

By observing the world around us, we see that everything is constantly influencing everything else (Maharishi Mahesh Yogi, 1969, p. 63). Each member of a family directly influences the feelings of every other member; no wave on the ocean is independent of any other. Maharishi points out that this principle of relationship or interconnectedness reaches its ultimate, infinite value in the transcendental level of Nature's functioning, pure consciousness, the field of "infinite correlation" where "every point in the field is infinitely correlated with every other point so there is no resistance in moving from one point to the other" (Maharishi Mahesh Yogi, 1995c, p. 287; see also 1995b, pp. 374, 389; *Maharishi's Master Plan to Create Heaven on Earth*, in press, pp. 277, 328–330). Each point in the unbounded field of pure consciousness is related with all other points so intimately that one is the other: "an impulse anywhere is an impulse everywhere." Furthermore, the communication among all points is described as perfect, meaning that it is instantaneous, complete, and a "frictionless flow."

Physics has *glimpsed* the quality of infinite correlation in the perfect connectedness of two distant particles, or any distant parts of a quantum mechanical system described by a single wave function. This phenomena, called the Einstein, Podolsky, Rosen paradox, has been observed experimentally (see Bohm & Hiley, 1993, pp. 134–159). It explains how a measurement made on one part of a system instantaneously transforms the remaining parts of the system, no matter how far distant.

Maharishi has explained in different contexts that pure consciousness has these qualities: it is infinitely correlated, all knowing, "programmed" with the complete knowledge of Natural Law, mathematically precise, computes with infinite speed, and has infinite organizing power (Maharishi Mahesh Yogi, 1995a; 1995c, p. 287; Maharishi International University, 1990b, p. 3; Nader, 1995, pp. 36–39). Due to these qualities, it is reasonable to conclude that pure consciousness has the properties of an ultimate computer. In this regard, Maharishi has referred to pure consciousness as the "cosmic computer" (Maharishi Mahesh Yogi, 1995a, p. 105; see Lester, 1987, pp. 313–316). The term cosmic, in this context, means omnipresent and omniscient. The cosmic computer spontaneously computes the innumerable transformations of Natural Law that are necessary to govern the entire universe at every point simultaneously.

Maharishi explains that once individual awareness gains access to pure knowledge and its infinite organizing power in higher states of consciousness, the human brain with its billions of neurons and interconnections becomes the hardware of the cosmic computer (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988). Through the perfect orderliness and coherent functioning of the brain, individual intelligence becomes cosmic intelligence allowing the student to spontaneously think and act in perfect accord with all the Laws of Nature. Complex situations are sorted out without lapse of time, and the knowledge necessary for one's success effortlessly comes to one's awareness. The cosmic computer functions effortlessly because it functions in accord with the principle of least action.

Principle of Least Action

The principle of least action, first formulated by Lagrange, Hamilton, and Jacobi in the 18th century, states that Nature generally functions so that the amount of action is least (Domash, 1974, p. 149). In all the mechanical motions of nature—the movement of the stars and planets, ocean waves, the propagation of light—effort is minimized and efficiency or economy is maximized. For example, when a child throws a ball, it spontaneously takes that trajectory which allows it to fly the greatest possible distance given all the conditions that affect its flight. All variables—gravity, speed and direction of the wind, weight and size of the ball, how hard the ball was thrown, weather conditions, air density—are taken into account and the optimum path is computed instantly. Through rigorous, computer-aided analysis, it becomes apparent that the ball could not take any other trajectory and fly as far. Less efficient paths are not strictly forbidden, but their probability of occurrence is infinitesimally small. Natural Law functions most efficiently and involves least effort because it "always chooses the simplest, most direct path to accomplish anything" (Maharishi Mahesh Yogi, 1995c, p. 305).

At the level of self-referral consciousness, Maharishi explains that action is least and achievement is greatest. This means that everything is achieved without any action, without any effort at all: "You can accomplish everything by interacting with your own Self. When you interact with your own Self, this is the field of silent performance" (Maharishi Mahesh Yogi, 1993, pp. 12–13; 1992, pp. 14–15). In the same way that a powerful businessman can create swings in the stock market just by a few quiet suggestions, the self-interacting dynamics of pure consciousness are able to create enormous changes throughout the universe through their silent transformations within consciousness (Maharishi Mahesh Yogi, 1995c, pp. 170–173; 1963, pp. 150–179). Thoughts and actions performed by the enlightened—those established in self-referral consciousness—have the infinite organizing power of Natural Law at their basis and, therefore, easily meet with success without creating problems for the performer. This idea is expressed in Ṛk Veda:

> यतीनां ब्रह्मा भवति सारथिः
>
> *Yatīnāṁ Brahmā bhavati sārathiḥ*
> —Ṛk Veda, 1.158.6
>
> Those established in the silent singularity of self-referral consciousness, motivate the infinite organizing power of Natural Law to be their charioteer.
> *(Maharishi Mahesh Yogi, 1995c, p. 172)*

By thinking and acting from within self-referral consciousness, one gains the support of Natural Law and intentions are accomplished easily, "leaving the doer fresh enough to enjoy fully the fruits of his action" (Maharishi Mahesh Yogi, 1969, p. 143). Moreover, one "begins to enjoy the results immediately." With reference to mathematics, this infinitely powerful skill is called "mathematics without steps."

Mathematics Without Steps

Maharishi's Vedic Mathematics is an enormously rich field of experience and understanding. One of the most fascinating characteristics of Vedic Mathematics described by

Maharishi is that it is "the commander of Natural Law" which "spontaneously designs the source, course, and goal of Natural Law" within the student's self-referral consciousness (Maharishi Mahesh Yogi, 1995b, pp. 335, 338). The mathematics and the reality structured by the mathematics occur simultaneously within the student's consciousness. This idea might be explained by an example which compares Vedic Mathematics with modern mathematics. Using modern mathematics, a physics class may calculate the trajectory, velocity, and distance of balls thrown by the students but only before or after the act of throwing. There is some passage of time between making the calculations and throwing the ball. However, at the highest level of Maharishi's Vedic Mathematics—mathematics without steps—the flight of the ball and the mathematical analysis and synthesis occur simultaneously within the student's awareness. This is because Maharishi's Vedic Mathematics is the quality of infinite organizing power inherent in self-referral consciousness that structures all activity in the universe, and infinite organizing power resides in the student's Self (Maharishi Mahesh Yogi, 1995b, pp. 338–341). Maharishi expresses this integrated nature of Vedic Mathematics by saying that it "is itself the mathematician, the process of deriving results, and the conclusion" and, furthermore, that the cognition of his Vedic Mathematics "is available to fully alert consciousness—*Ṛitam-bharā pragyā*; it is available within the *Ātmā* of everyone, in the self-referral consciousness of everyone."

The seat of Vedic Mathematics is in self-referral consciousness where Ṛishi, Devatā, and Chhandas first emerge from within Saṁhitā. This point of emergence is further described as "the seat of all precision and order in the universe," that "level of Creative Intelligence from which all the number systems and the mathematical structures systematically and sequentially emerge" (Maharishi Mahesh Yogi, 1995b, pp. 365, 340). Maharishi explains that the absolute precision of Vedic Mathematics first designs the structuring dynamics of the Veda, and then the sequential structure of the Vedic Literature, and then the orderly structure of the ever expanding universe while simultaneously maintaining the connectedness of each aspect to self-referral consciousness:

> Vedic Mathematics is the self-regulating, self-perpetuating, self-sufficient precision tool of self-referral consciousness which spontaneously creates the structuring dynamics (Vedic Literature) from within the one unbounded ocean of self-referral consciousness, and continues to create the holistic structure of the Veda, and from within this holistic structure of the Veda (Ṛk) continues to create the specific sequentially developing structures of the Veda, and from this, following the same laws of evolution, Vedic Mathematics, functioning from within the structure of the Veda, stimulating the Creative Intelligence of the Veda, stimulating the organizing power of the Veda, creates the material universe and with absolute precision of evolution, the ever-expanding universe continues to emerge.
>
> In this ever-evolving procedure of evolution, the perfection of the all powerful, invincible element of Vedic Mathematics manages every step of evolution to remain connected with the infinite source of energy and intelligence in the self-referral state of the holistic value of precision and order in self-referral consciousness. (1995b, pp. 354–355)

From this quote, it becomes apparent that Maharishi's Vedic Mathematics remains entirely within self-referral consciousness, the Self.

We have seen that a student's experience in Brahman Consciousness is that everything in the universe, the totality, is nothing other than the Self, *Ātmā*. This experience is expressed in the Upanishads:

अहं ब्रह्मास्मि

Ahaṁ Brahmāsmi

—Bṛihad-Āraṇyak Upanishad, 1.4.10

I am Totality. *(Maharishi Mahesh Yogi, 1995b, p. 555)*

अयम् आत्मा ब्रह्म

Ayam Ātmā Brahm

—Māṇḍūkya Upanishad, 2

The pure, silent, simple singularity of Ātmā is the Totality—Brahm. *(Maharishi Mahesh Yogi, 1995b, p. 557)*

Based on the experience of these affirmations, Maharishi explains that the scope of his Vedic Mathematics ranges from the individual Self, *Ātmā*, to the cosmic Self, *Brahm* (Maharishi Mahesh Yogi, 1995b, pp. 359–360). So from the most developed level of experience and understanding, Maharishi's Vedic Mathematics remains in the Self and analyzes and synthesizes the orderly design of the entire universe. This orientation within the Self contrasts with the logical progression of modern mathematics.

Maharishi explains that modern mathematics has started from the field of diversity and moved in logical steps towards its abstract, unified source (Maharishi Mahesh Yogi, 1995b, p. 384). This is expressed in physics. Like the other disciplines of modern science, physics utilizes mathematics to express in precise formulas and numerical values all the logical steps of progression from the concrete field of diverse phenomena toward the source of all diversity in the abstract, Unified Field of Natural Law. In contrast, Maharishi points out that his Vedic Mathematics remains in self-referral intelligence and deals with the whole structure of diversity from within that unified state:

> Modern Mathematics is the field of steps, whereas Vedic Mathematics is the field of pure intelligence that gets what it wants instantly without steps. It is the field of infinite correlation, the field of simultaneity of all steps, because it functions in the frictionless field of infinite correlation—the field of self-referral intelligence. In Vedic Mathematics all steps are synthesized to promote the result without the need for going through the steps and stages to arrive at the goal. (1995b, p. 389)

Vedic Mathematics has its reality in self-referral consciousness where knower, knowing, and known are infinitely correlated. In this unified state of knower and known, knowledge is a spontaneous revelation, not a step-by-step derivation. Solutions occur at the same time as the problems. *Brahmā bhavati sārathiḥ*, the infinite organizing power of Natural Law spontaneously works "for him who is established in balance, evenness—functioning through the Principle of Least Action" (Maharishi Mahesh Yogi, 1995a, p. 175). Since the logical steps of problem solving or calculation are not needed to successfully achieve any goal, Maharishi refers to this level of life as "mathematics without steps" (Maharishi Mahesh Yogi, 1995b, pp. 389, 627, 632; 1991; 1990).

Maharishi (1990) explains that mathematics without steps is the highest level of his Vedic Mathematics, the ability to know anything and do anything spontaneously through the instrument of Natural Law:

> So the supreme level of Vedic Mathematics gives the Vedic Mathematician the ability to spontaneously compute all the Laws of Nature in his awareness, so that the sprouting and growth of a desire into action and achievement is all conducted spontaneously by the Laws of Nature.

Here, individual intelligence has become completely identified with universal intelligence and gains infinite organizing power. The Vedic Mathematician's perfectly coherent brain becomes the cosmic computer capable of spontaneously organizing all the Laws of Nature to fulfill a desire. Maharishi teaches that the mathematician in this state is like a catalyst: the student's consciousness remains involved with itself in self-referral consciousness and yet stimulates precise activities in the environment which resolve problems spontaneously and promote health and well being (Maharishi Mahesh Yogi, 1995b, p. 356). This emphasis on spontaneity indicates that the catalytic influence has its effect without friction and with infinite speed.

Maharishi (1991) comments on the state of consciousness which gives rise to a mathematics without steps:

> On one level, whenever there is a problem, there is a solution. There are no steps between the two. That means mathematics without steps. You do not have to solve a problem through many steps. This is the supreme level of mathematics and the supreme level of wakefulness of human consciousness, of Transcendental Consciousness, where the Vedic Script, the Constitution of the Universe,[4] is written and from where all the Laws of Nature emerge and conduct the activity in Nature. That is the instantaneous solution to any problem. That is one level of Jyotish which is the level of the unified field in terms of modern science. It is Yogic consciousness, singularity of consciousness, the fully awake, self-referral level of consciousness. You do not work out what you want to know. It's there. It's all knowingness.

Jyotish is one of the six Vedāngas of Vedic Literature. In terms of Jyotish, the "all knowing, omnipresent pure intelligence" is called *Jyotishmatī pragyā* (Maharishi Mahesh Yogi, 1995b, p. 376). Maharishi explains that the purpose of Jyotish is to structure *Jyotishmatī pragyā* in human awareness enabling the student to spontaneously engage in the infinite organizing power of Natural Law, to begin to function through the principle of least action as "the absolute manager" or "Cosmic Mathematician" who successfully works out the orderly process of creation and evolution of everyone and everything in the entire universe. Maharishi goes on to point out that the Cosmic Mathematician is present within everyone as the field of pure knowledge, the Veda, with infinite organizing power:

> The Cosmic Mathematician is present within everyone; it is the light of life of everyone; it is present within everyone in the structure of Ṛk Veda and the Vedic Literature; it is the light of pure intelligence. Human awareness, gaining its simplest state, most natural state, comes in full alliance with this Cosmic Mathematician, and actually becomes it—becomes *Ritambharā pragyā*, all-knowing intelligence—becomes the Cosmic Mathematician, and from that level always functions as the grand custodian of Natural Law enabling every aspect of daily life to be lived free from mistakes—every aspect of private and professional life always in the evolutionary direction of Natural Law enjoys the dignity of perfection. (1995b, pp. 376–378)

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[4]Just as the constitution of a nation represents the most fundamental level of national law and the basis of all the laws governing the nation, Maharishi explains that the Veda is the "Constitution of the Universe" because it is the most fundamental level of Natural Law and the basis of all the Laws of Nature that govern the entire universe (Maharishi Mahesh Yogi, 1995a, pp. 203–213; Wallace, 1993, p. 234).

Human awareness, becoming fully established in its simplest state, gains "the fruit of all knowledge," the ability "to know anything, do anything spontaneously right, and achieve anything through the support of Natural Law" (Maharishi Mahesh Yogi, 1995a, pp. 20–28). The achievement of this infinitely powerful, holistic problem-solving skill ideally should be the ultimate goal of mathematics education.

However, mathematics education is generally unaware of consciousness—the field of pure knowledge, the Veda—and its practical, applied value. So mathematics teachers are unable to connect mathematics to its fundamental basis in Natural Law or empower students with its infinite organizing power. Without access to pure knowledge, mathematics students' are unable to develop their full mental potential or achieve the most powerful level of problem solving. This fundamental lack has produced incoherent, frustrated students unable to live life free from mistakes. Ṛk Veda (1.164.39) expresses this perspective, *Yastanna veda kim ṛichā karishyati ya it tad vidus ta ime samāsate*, which Maharishi translates as, "he who does not have self-referral consciousness is full of mistakes—he who is not established in self-referral consciousness does not know how to think spontaneously, mathematically right" (1995b, p. 349). However, those established in self-referral consciousness gain evenness, wholeness of life—life in perfection without mistakes.

We have seen that self-referral consciousness is the "cosmic computer" which computes the time evolution of the entire universe without a problem. The all comprehending, mistake-free computations of Natural Law occur in a mathematically precise sequence due to the absolutely precise organizing power inherent in the structure of pure knowledge, the Veda. This precise mathematical sequence of the Veda is known as Maharishi's *Apaurusheya Bhāshya*, "the total display of Vedic Mathematics" (Maharishi Mahesh Yogi, 1995b, p. 370, 621–622, 635–638; Nader, 1995, pp. 36–39; Muehlman, 1994, pp. 120–129; Wallace, 1993, pp. 234–250; Dillbeck, 1989, p. 127–128). It is the highest level of Maharishi's Vedic Mathematics—wherein the knowledge and the reality it describes are identical, the cognizer and the mathematics cognized are the same entity. It is the mathematics of the Veda, the structuring dynamics of self-referral consciousness.

We have also discussed the practical value of this state which one realizes through the development of higher states of consciousness. In Brahman Consciousness, *Jyotishmatī pragyā*, individual intelligence, becomes universal intelligence and the student gains all knowingness, the infinitely powerful administrative skill called mathematics without steps. Being fully aligned with universal intelligence, the student is the Cosmic Mathematician with infinite organizing power. Whatever the student desires to know is instantly available on the level of the awareness. This discussion elaborates the most important characteristic of Maharishi's Vedic Mathematics: it is always connected to consciousness, always a meaningful, living reality capable of transforming the life of the mathematician into higher states of consciousness.

Even though we have just seen a vision of ideal mathematics, Maharishi explains that there is another level of Vedic Mathematics (Maharishi Mahesh Yogi, 1995b, p. 393). This secondary level provides formulas for calculating the desired result through a sequence of steps. An example of this form of Vedic Mathematics is Vedic Sūtra based computation.

Introduction to Vedic Sūtra Based Computation

Vedic Sūtra based computation is derived from sixteen *Sūtras*[5] or aphorisms from Atharva Veda (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988). An example is the *Sūtra* ऊर्ध्वतिर्यग्भ्याम् *Ūrdhva-tiryagbhyām*, which means "vertically and crosswise" and is used in multiplication (Tirtha, 1965, p. 40). These *Sūtras* or formulas of Natural Law derived from the Veda, distill the sequential steps of an algorithm[6] into one central, key point that is easy to remember. Examples of how Vedic *Sūtras* give the concise, central point of an algorithm are shown in Table 1. Maharishi refers to the *Sūtras* as "refined formulas" that "reset the brain" so that it operates in accord with Natural Law as one computes (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988; B. Morris, fax communication to Maharishi University of Management, 1988). This activity cultures the ability of the mind to think and act from the level of pure consciousness, as one works through a specific mathematical problem.

In recent times, these *Sūtras* were discovered and practically applied by Swāmī Tirtha, past Shankarāchārya of Govardhana Math at Puri, India. Some of the algorithms presented by Tirtha (1965) appeared in earlier epochs. For example, the general multiplication algorithm based on the *Ūrdhva Sūtra* has been described in writings from the 16th century (Cajori, 1928, p. 263; 1897, p. 146). Datta and Singh indicate this method was known to Indian scholars of the 8th century or earlier (Datta & Singh, 1935, pp. 145–146). However, the *Ūrdhva Sūtra* which governs the method and its relationship to consciousness were not mentioned by either source. This suggests that only the surface value of Vedic Mathematics was known and used by the mathematicians of that era.

On the surface, each *Sūtra* provides the essence of an algorithm used to perform a calculation. However, Maharishi emphasizes that the *Sūtras* have a deeper value which is predicted to make the brain physiology more orderly and coherent:

> On one level, each *Sūtra* aphoristically describes the actual computational method to be used for the various types of mathematical problems. On a deeper level, however, each *Sūtra* may be seen as a refined formula for producing a high degree of coherence and order in brain functioning, facilitating the rapid and precise solution to the mathematical problem. (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988)

This quote indicates that it is the increased order and coherence that makes the act of computing more accessible and smooth for the student. The following discusses how the *Sūtras* enliven Natural Law and gradually develop consciousness in the direction of cosmic computing.

As with the other Vedic Technologies taught by Maharishi, instructions regarding how to use the *Sūtras* are personal and private (Maharishi Mahesh Yogi, 1963, p. 52). However, Maharishi does explain how the *Sūtras* enliven Natural Law in one's awareness:

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[5] For a complete table of the *Sūtras* and their meanings as well as a compilation of Vedic Sūtra based algorithms, see Muehlman, 1994, pp. 138–144, 281–343, 351–362.

[6] An algorithm is a systematic, step-by-step procedure used to effectively solve a problem. Examples are shown in Table 1.

Table 1

Elementary Examples of How Vedic *Sūtras* Provide the Key to Their Algorithms

Addition using the *Shuddhah Sūtra*, meaning "purified" (Puri, 1986, p. 34; N. Puri, personal communication, 1990):

Example: 93 + 28 + 65

Algorithm: *Shuddhah* addition follows this procedure.

1) When the sum reaches 10 or more, purify 10 out of the sum (take away 10) and note it as a dot.
2) Addition continues with the remaining amount.
3) The number of dots in a column is the amount to be carried.

```
  9• 3
  2  8•
+ 6  5
1 8  6
```

Beginning in the ones column, 3 + 8 = 11. Take away 10 from the sum and note the transaction as a dot. Continue adding with 1: 1 + 5 = 6. Record the 6. There is 1 dot in the ones column so carry 1. In the tens column, 1 + 9 = 10. Record the 10 as a dot and continue adding: 2 + 6 = 8. There is 1 dot in the tens column which is carried to the hundreds place of the answer: 186.

Notice how the *Sūtra* provides the key procedural element of the algorithm: purifying 10 out of the mental sum.

Subtraction with the *Shuddhah Sūtra*, which means "purified" (Puri, 1986, p. 39; N. Puri, personal communication, 1990):

Example: 5125 – 3608

Algorithm: *Shuddhah* subtraction follows this procedure.

1) If the lower digit is less than or equal to the upper, subtract normally.
2) If the lower digit is larger than the upper digit, then:

A) Write a dot beside the next digit in the lower portion of the problem. The dot is a reminder to increase that digit by 1 and thereby purify 10, or a power of 10, out of the upper portion of the problem (minuend).

B) Add the complement of the lower digit to the upper digit and record the sum.

```
   5  1 2  4
 – 3•6 0•8
   1  5 1  6
```

Beginning in the ones column, 8 is larger than 4 so write a dot near the 0. Add the complement of 8 to the four: 2 + 4 = 6. In the tens column, the dot beside the 0 reminds the student to add 1 to the 0. One is less than 2 so subtract normally: 2 – 1 = 1. In the hundreds column, 6 is greater than 1 so write a dot near the 3. Add the complement of 6 to the 1: 4 + 1 = 5. In the thousands column, 4 is less than 5 so subtract normally: 5 – 4 = 1.

The *Sūtra* provides the key procedural element of the algorithm: purifying 10, or a power of 10, out of the upper portion of the problem (minuend).

Table 1 (continued)

Multiplication with the *Ūrdhva Sūtra*, meaning "vertically and crosswise" (Tirtha, 1965, p. 40; also see Tables 5 and 6):

Example: 27 x 34

Algorithm: The following procedure is for problems of two digits multiplied by two digits.

1) Multiply vertically in the ones column: (4x7) = 28. Record the 8 and carry the 2. 2) Multiply crosswise and add the two products: (4x2)+(3x7) = 8+21 = 29. Add the carry, 29+2=31. Record the 1 and carry the 3. 3) Multiply vertically in the tens column: (3x2) = 6. Add the carry, 6+3=9. The answer is 918.	2 7 <u>x 3 4</u> 9/1/8 3 2

In algebra: $(ax+b)(cx+d) = x^2(ac) + x(ad+bc) + bd$ where $x = 10$

Notice how the entire algorithm unfolds from the *Sūtra's* meaning.

Special case multiplication when both factors are near a base of 10 (i.e., a power of 10) using the *Nikhilam Sūtra*, meaning "all from nine and last from ten" (Tirtha, 1965, pp. 13–29):

Example: 92 x 97

Algorithm: When applying *Nikhilam Sūtra*, all digits are subtracted from 9 except the last one (the ones place digit in this example) which is subtracted from 10.

1) Apply *Nikhilam Sūtra* to the factors:
For 92, 9–9=0 and 10–2=8 so the deviation from the base of 100 is –08.
For 97, 9–9=0 and 10–7=3 so the deviation from the base is –03.
Now rewrite the problem as shown:

92 –08 <u>x 97 –03</u> 89 / 24 = 8,924	2) The right-hand side of the answer is found by multiplying the deviations: –03 x –08 = 24. 3) The left-hand side of the answer is found by adding diagonally in either direction: 92+(–03)=89 or 97+(–08)=89. The answer is 8,924.

In algebra: $(x-a)(x-b) = x(x-a-b)+ab$ where "x" is the base of 100 and "a" and "b" are deviations from that base

The deviations found by applying *Nikhilam Sūtra* are the key to making the algorithm work. This shows how the *Sūtra* provides the procedure which is pivotal to the algorithm.

Table 1 (continued)

Division with *Dhvajānkam Sūtra*, meaning "on top of the flag" (Tirtha, 1965, p. 240):

Example: 8547 ÷ 24

Algorithm: The following procedure is for problems with two-digit divisors.

Rewrite the problem with the divisor's right digit higher than its left. This raised digit becomes "the flag digit" and will be used to multiply digits in the quotient. The left digit is used to divide. Once this distinction in the divisor is made, the algorithm unfolds:

1) Divide. Divide and record the result as a quotient digit. Write the remainder as a subscript to the left of the next digit in the number to be divided. These two form a new dividend. If this quotient digit is not directly underneath the right-most digit of the original dividend, continue.

> Look ahead: While dividing, look ahead to see if the answer in Step 3 is going to be negative. If so, decrease the quotient digit until the answer in Step 3 is positive. This process simplifies computation. This step of looking ahead is not necessary if the student is practiced with the Vinculum method (see Appendix) and can rely on it.

2) Multiply. Multiply the quotient digit by the flag digit.

3) Subtract. Subtract the product from the new dividend. Return to Step 1.

Example:

$2^4 \div 8$	${}_2 5$	${}_3 4$	: ${}_2 7$		New dividends
	-12	-20	-24		Quotient digit times the flag-digit (to be done mentally if comfortable)
	13	14	3		
3	5	6	.	1	Quotient digits

Elevating the right digit of the divisor so that it appears like a flag provides a graphic mnemonic device. It also prepares the student to compute with the two divisor digits in different ways. This shows how the *Sūtra* sets the stage for the entire algorithm to unfold.

Table 1 (continued)

Special case division when the divisor is near a base of 10 (e.g., 8 or 96) using *Nikhilam Sūtra*, "all from nine and last from ten" (Tirtha, 1965, p. 55; Puri, 1988b, p. 96):

Algorithm: When applying *Nikhilam Sūtra*, all digits are subtracted from 9 except the last one—the digit in the ones place—which is subtracted from 10.

1) Apply *Nikhilam* to all digits in the divisor. This gives the modified divisor (MD). Write the MD under the original divisor.

2) Bring down the first (left most) digit of the dividend as the first digit of the quotient and as the sum of column one. Multiply it by the MD. Write the product beneath the next digit of the dividend.

> Look ahead: The sums of each column should be small so that multiplication by the MD is easy. The Vinculum method (see Appendix) may be applied to the dividend or to the products if the student has practiced with it and can rely on it.

3) Continue the process of adding the columns and multiplying the sum by the MD until the last digit of the product is under the last digit of the dividend.

Example: 125 ÷ 8

Apply *Nikhilam Sūtra* to the divisor: 10 – 8 = 2. The MD = 2.	8 ÷ 1 2 / 5 2 2 8 1 4 / 1 3
Since the remainder is larger than the divisor, divide the remainder and add the result to the quotient.	8 ÷ 1 / 3 2 2 1 / 5
	1 4 + 1/5
The answer is:	15 R5 (R=remainder)
For a decimal remainder, divide the fractional remainder:	/ 5 0 0 10 20 / 5 0 0 1 2
The answer is:	1 5. 6 2

This example shows how *Nikhilam Sūtra* provides the basic or key element, the MD, which the algorithm returns to again and again.

> When the *Sūtras* of Vedic Mathematics are present in the awareness, one gets into a very generalized and graphic state of awareness. If then brought to a specific problem, this generalized awareness becomes focused on the parts. This cultures the ability of the mind to function from its unbounded, cosmic level when it is applied to a specific mathematical problem. This brings harmony with Natural Law, perfect accord with all the Laws of Nature. (Maharishi Mahesh Yogi, private communication with M. Weinless, 1988; see also Brooks & Brooks, 1988)

Here, Maharishi explains that the *Sūtras* culture the ability to think and act in accord with Natural Law so that the mind is operating in the proper sequence as it computes. Thus, students begin to think more in accord with all the Laws of Nature as they apply the *Sūtras* to computation.

When the *Sūtras* are used as instructed, Maharishi states that the human brain gradually begins to exhibit the characteristics of the cosmic computer:

> At this level, the brain with its billions of neurons and interconnections, is to be regarded as the "cosmic computer," which through proper programming can accomplish any result. The *Sūtras* of Vedic Mathematics are the "cosmic software" that create the ability to compute most rapidly and precisely. (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988)

As cosmic software, the *Sūtras* of Vedic Mathematics are the self-interacting dynamics of consciousness which culture the internal transformations of consciousness so that the individual's awareness begins to compute in a natural, coherent manner.

Further, Maharishi points out that the *Sūtras* are instrumental in developing mathematics without steps (Maharishi Mahesh Yogi, private communication with Dr. Bevan Morris, 1991). By culturing the ability of the mind to function from its unbounded level while it computes in accord with Natural Law, the *Sūtras* gradually, yet systematically, develop the ability to think in perfect harmony with Natural Law. The *Sūtras* thus provide an important technology for the development of higher states of consciousness.

As the ability to think and act in accord with Natural Law grows, it is predicted that computation becomes faster, easier, and increasingly self-referral (Indian Institute of Maharishi Vedic Science and Technology, fax communication to Maharishi University of Management, March 1988; Maharishi Mahesh Yogi, private communication with Dr. Bevan Morris, 1991). To become increasingly self-referral means that computation occurs more within consciousness and is less dependent on external means such as paper-and-pencil or calculator. Increased mental mathematics may be an interim stage of this developmental process. The following section presents a literature review on Vedic Sūtra based computation as applied in a variety of educational settings.

Literature Review: Characteristics of Vedic sutra based Computation Relevant to the Experimental Study

The following literature review discusses only the points relevant to the research experiment which follows. Additional points descriptive of the entire field of Vedic Mathematics are reviewed in the Appendix. Understanding Vedic Mathematics is like learning a new language; one needs to approach it with a receptive, open mind to gain its full value:

> The western-trained mathematician is cautioned against trying to fit this system into the mental framework he already has, which is what most western training encourages us to do. If the

> methods are practiced with a neutral attitude of mind, allowing the system to speak for itself, then the possibility is opened of entering the spirit of this approach. Otherwise there is a risk of simply acquiring a few techniques, and not really gaining any overall sense of the approach. (Nicholas, Williams, & Pickles, 1984, p. xi)

The points reviewed here include references to the efficiency and ease of the Vedic Methods, the ease of learning Vedic Mathematics, the flexible format for recording the steps and answer of a calculation, the conduciveness to mental computation, and the enjoyment that comes from this approach to computation.

Efficient Algorithms and Ease of Learning

Tirtha notes that "in some very important and striking cases, sums requiring 30, 50, 100 or even more numerous and cumbrous 'steps' of working using the current Western methods can be answered in a single, simple step of work by the Vedic method" (Tirtha, 1965, p. xvii, xxi). For example, the answer to the problem 1/39 = 0.025641 may be easily worked on one line in less than 10 seconds using the *Sūtra Ekādhikena pūrveṇa*, "by one more than the previous one" (Puri & Weinless, 1988, p. 3; Puri, 1988a, p. 77).

Even for complex problems, Tirtha states that "the time taken by the Vedic method will be a third, a fourth, a tenth or even a much smaller fraction of the time required according to modern Western methods" (Tirtha, 1965, p. xvii). Puri concurs that the Vedic Methods require less time and effort (Puri, 1991, p. 16). Due to its efficiency, many students and adults find that Vedic Mathematics saves time and energy when used on timed tests, professional exams, or during competitions. Finally, Tirtha estimates that by using the Vedic sutra based approach to mathematics education, the whole course of mathematics taught in Western universities may be learned in about one-third to one-tenth the time (Tirtha, 1965, p. xvii).

Sometimes adults learning Vedic Sūtra based computation say that it is more difficult or that it has more steps than conventional computation. In my experience, and that of Continuing Education teachers at Maharishi University of Management in Fairfield, Iowa, the reason most often given for preferring western methods is that adults have become accustomed or conditioned by their previous education. Most adults have spent 20 or more years adapting their minds to the conventional methods used to add, subtract, multiply, or divide. By the time they experience Vedic Sūtra based computation, it may be that the conventional algorithms have become so familiar that they seem more logical and natural than the Vedic Methods, even though some evidence exists to suggest the opposite.

Flexible Format

The format used for recording the intermediate steps of computation is highly flexible (Puri, 1991, pp. 2, 11–13). For the *Ūrdhva* method of multiplication, Puri notes that the student may do as much or as little of the computation mentally as desired (Puri, 1986, pp. 63–71). The steps of the *Ūrdhva* method of multiplication are briefly described in Table 1 and analyzed in more detail later in this section.

Depending on how much mental arithmetic the student would like to do, the *Ūrdhva* method may be worked in one, two, or three lines. In the former, all the carries are done mentally, and in the latter, they are added as the final step. Also, the flexible format of the *Ūrdhva* method allows the student to either write out the intermediate steps of multiplication and addition in the space around the parent problem or do them mentally. Finally, the format allows one to start computations either from the left or from the right (Nicholas, Williams, & Pickles, 1984, pp. 7–8). By calculating the left digit first, one may obtain the most significant digit in the first calculation and continue calculating the digits to the right until the desired level of accuracy is reached (Puri, 1991, p. 14).

Conducive to Mental Computation

Mathematics educators and researchers indicate that mental computation is one of the best ways of helping children become independent of memorization techniques and gain deeper insights into the number system (Hope, 1987, 1986; Williams, 1991, 1984; Trafton, 1978; Beberman as cited in Sister Josephina, 1960). Trafton points out that mental arithmetic adds a new dimension and vitality to computation, increases flexibility, gives new insights into numbers and number relationships, and develops problem-solving skills (Trafton, 1978, p. 200). Williams notes that "mental calculation sharpens the mind and increases mental agility and intelligence" which he believes is evident to anyone who has practiced or taught mental calculation (Williams, 1991, p. vi). He goes on to say that in mental computation,

> the subtle properties of numbers and their relationships are appreciated much more readily than if the calculation was written down . . . the intrinsic qualities, relationships, and beauty of numbers and the way they create new numbers out of themselves is a source of great enjoyment. (Williams, 1991, p. vi)

In spite of its value, mental computation is not developed enough in the classroom (Cockcroft, 1982; Trafton, 1978; National Council of Teachers of Mathematics, 1986). Mathematics education generally teaches one basic approach to each operation and relies too heavily on paper and pencil exercises. The rigid, rule-bound approaches to computing Hope observed "suggests that children are enslaved to a technique and that their alternative methods of solution are not known or they have been rejected by the user as being somehow 'improper'" (Hope, 1986, p. 51). Perhaps it is due to this rigid approach that only 55 percent of 17-year-olds were able to multiply numbers like 90 and 70 in their head (National Assessment of Educational Progress, 1983, p. 32).

Williams (1991) believes that the reason for the lack of mental mathematics today is the complexity and unrelated nature of the formulas of modern mathematics. However, he explains that Vedic Mathematics actually encourages mental mathematics due to the naturalness and coherence of its techniques:

> Vedic Mathematics provides a coherent structure for mathematics: the Vedic methods are beautifully interrelated and complementary. While modern mathematics is a hodgepodge of unrelated techniques, bewildering in their complexity, the Vedic system offers unifying and natural principles whose effect is to transform mathematics into an easy and delightful activity. (Williams, 1991, pp. ix–x)

Williams goes on to say that Vedic Methods are conducive to mental mathematics for another reason: it provides algorithms like the *Ūrdhva* method of multiplication that may be solved on one line without paper and pencil.

Upsurges of Joy

Puri points out that the naturalness and ease of Vedic Sūtra based computation "brings smiles on the face and joy in the heart" of the students (Puri, 1986, p. 4). Further, Vedic Mathematics reduces anxiety and increases playfulness:

> The very natural, easy, and super fast algorithms of Vedic Mathematics bring an upsurge of joy. . . . Due to its very special and universal features, the system of Vedic Mathematics converts dry and tedious maths into a playful and joyful subject which children learn with smiles. Therefore Vedic Mathematics is the gift of the Veda to solve the problem of mathematics anxiety being faced by mathematics education in the whole world. (Puri, 1986, p. 8)

At Maharishi School in Fairfield, Iowa, children sometimes laugh and experience "thrills of bliss" while solving a problem with the Vedic Sūtra based approach (see Table 3). Compared to conventional computation, teachers usually report that their students enjoy Vedic Mathematics more and are more motivated by it. These impressive qualities of Vedic Mathematics inspired a series of pilot studies that served as precursors to a more rigorous, empirical study.

Pilot Research on Vedic Sutra Based Computation

Several pilot studies on the attitudes and performance of students using Vedic Sūtra based computation were conducted at Maharishi School of the Age of Enlightenment in Fairfield, Iowa. These studies were used to determine the best areas of Vedic Sūtra based computation for controlled research. Encouraging trends were found in five significant areas: affect and attitudes, information processing, achievement, mental mathematics, and checking.

Pilot Study Number One

This was an exploratory survey of attitudes and perceptions about Vedic sutra based computation among students and teachers. The subjects were ninety-three 3rd–5th grade students and the six teachers who taught them. Separate surveys were constructed for the teachers which included essay responses. The surveys were administered in the spring of 1989 after the students had been introduced to several of the Vedic Methods. All subjects learned Vedic Sūtra based addition and subtraction based on *Shuddhah Sūtra* and Vedic Check, based on *Gunita-samuchchayah samuchchaya-gunitah Sūtra*. Grades 4–5 also learned Vedic Sūtra based multiplication based on *Ūrdhva Sūtra* and Vinculum based on *Nikhilam Sūtra*. Depending on their skill level, these students also learned for the first time, practiced, or mastered the conventional methods of addition, subtraction, and multiplication. The surveys were given once, after all the Vedic Methods had been introduced.

Results from the survey in the area of affect and attitude indicated that 78 percent of the students agreed they liked Vedic Sutra based computation, 53 percent said it made them happy, and 45 percent said it made them blissful. Seventy-two percent of the students agreed with the statement, "Since I learned Vedic Mathematics, I am more interested in all of mathematics," while 57 percent agreed with the statement, "Vedic Mathematics has made me more confident with all of mathematics" (see Table 2). These results were supported 100 percent by the teachers who said that "my class is enthusiastic about Vedic Mathematics," "my class enjoyed Vedic Mathematics more than regular mathematics," and

Table 2
Student Attitudes and Affect toward
Vedic Sūtra Based Computation—Pilot Study Number 1

	Percent strongly agreeing or disagreeing			
	3rd grade	4th grade	5th grade	Weighted average
Attractiveness	n=19	n=48	n=26	n=93
• Vedic Mathematics makes me happy.	79	56	27	53
• Vedic Mathematics is easy.	84	83	65	78
• I like Vedic addition.	84	83	77	83
• I like Vedic subtraction.	79	67	35	60
• I like Vedic multiplication.	79	77	85	80
• I like Vedic Check.	95	85	73	84
Process and Discovery				
• When I use Vedic Mathematics, I do more math in my head than when I use regular mathematics.	63	48	58	54
• Vedic Mathematics has taught me that there is more than one way to do a problem.	89	96	100	96
• Since taking Vedic Mathematics, I find more patterns and relationships among numbers.	95	92	92	92
• Since I learned Vedic Mathematics, I have discovered new ways of solving or checking a problem without my teacher's help.	74	58	65	63
Impact on Mathematics				
• Since taking Vedic Mathematics, I feel more comfortable with numbers.	89	83	77	83
• Since I learned Vedic Mathematics, I am more interested in all of mathematics.	84	68	69	72
• Vedic Mathematics has made me more confident with all of mathematics.	53	59	58	57
• I like Vedic Mathematics more than regular mathematics.	89	69	61	71

Table 3
Teacher Attitudes and Affect toward
Vedic Sūtra Based Computation—Pilot Study Number 1

Attractiveness	Percent strongly agreeing or agreeing (n = 9)
• Teaching Vedic Mathematics was fulfilling to me as a teacher.	100
• My class is enthusiastic about Vedic Mathematics.	100
• Vedic Mathematics is easy to teach.	100
Comparison	
• My students seem to enjoy Vedic Mathematics more than regular mathematics.	100
• My students checked their answers more frequently with Vedic Mathematics than with regular mathematics.	100
• In general, I like to teach Vedic Mathematics more than regular mathematics.	100
Involvement of Students	
• Since we began Vedic Mathematics, some of my students discovered new ways of solving or checking a problem without my help.	100
• A few students who were slower at mathematics became more inspired and improved more than expected when they were taught Vedic Mathematics.	100
• As a result of Vedic Mathematics, my students “played” with numbers more or felt more at home with numbers.	100
• My students became more involved with Vedic Mathematics than they did with regular mathematics.	100
Process	
• Vedic Mathematics has taught my students that there is more than one way to do a problem.	100
• Since we began Vedic Mathematics, my students discover more patterns and relationships among numbers.	100
• With Vedic Mathematics, my students do more math in their heads than with regular mathematics.	67

Table 3 (continued)

"Vedic Mathematics captures the students on a deeper level of interest and imagination. It touches some students that regular math does not." —3rd grade teacher

"The subtraction technique is so much easier to teach and use. Since you don't have to regroup and borrow, it is much easier. They loved turning a subtraction problem into an addition problem." —3rd grade teacher

"Vedic Mathematics enlivens the student's creativity with numbers. Children enjoy its playfulness and so enjoy math more." —4th grade teacher

"Everyone succeeds. There is no one way to relate to numbers so students feel free to attempt a variety of creative solutions. This manipulative thinking practice carries over into other areas of learning as well. Trying new ideas in new sequences helps them compare new relationships between a variety of factors forming totally unique concepts. This experience is very exciting." —4th grade teacher

"It's true that Vedic Mathematics is the math of smiles. The students would laugh and get excited when they would first use it to solve a problem. It's so fresh and easy."

—4th grade teacher

"The best part of Vedic Mathematics is the joyfulness the students feel when they use it, the 'aha' experiences, and thrills of bliss." —4th grade teacher

"It developed intuition with numbers and the ability to see mathematics everywhere in creation. Vedic Mathematics also helps transfer my enthusiasm for mathematics to my students." —5th grade teacher

"Vedic Mathematics opens mathematics education to exploration, making hypotheses and testing them." —Lower School Principal "as a result of Vedic Mathematics, my students played with numbers more or felt more at home with numbers" (see Table 3).

In the area of mental mathematics, 54 percent of the students agreed that when they use Vedic Sūtra based computation, they do more math in their head than when they use conventional mathematics (see Table 2). This result was confirmed by 67 percent of the teachers (see Table 3). These percentages are impressive considering that the curricula involved were not designed to improve mental computation. Furthermore, 92 percent of the students found more patterns and relationships among numbers (see Table 2), a finding which was confirmed by all of their teachers (see Table 3).

Regarding checking, students reported that they checked their work more than twice as often when they used Vedic Check as compared to when they used the conventional method of checking, and all of their teachers agreed, "my students checked their work more frequently with Vedic Mathematics than with regular mathematics" (see Table 3). Also, 63 percent of the students agreed they made less mistakes with Vedic Sūtra based computation than with conventional mathematics. This statement was confirmed by 67 percent of their teachers.

All the teachers agreed that teaching Vedic Mathematics was fulfilling and they wanted to learn more about it (see Table 3). Table 3 also presents several personal anecdotes by the teachers revealing the creative, joyful, intuitive, easy, and imaginative nature of Vedic Mathematics.

Pilot Study Number Two

The effect of Vedic Sūtra based computation on speed and accuracy of information processing was also evaluated. This study employed a one-group, pretest-posttest, within subject design (Gay, 1987, p. 281). In an attempt to identify those who were equally competent with both conventional and Vedic Methods of computing, students in grades four and five of Maharishi School took two matched achievement tests, each composed of subtraction and multiplication problems. The tests were given on separate days but at the same time of day. One test asked the students to use conventional methods of computation, and on the other they were asked to use the Vedic Methods. The Vedic Methods were based on the *Sūtras Shuddhah* for subtraction and *Ūrdhva* for multiplication. An attitude survey was also administered to identify students who liked the two methods equally and felt equally skilled in both. Further, the teachers of the respective classes were asked to identify those students who were equally skilled in both methods. Based on the results of these three qualifying steps, students who liked both the conventional and Vedic methods equally and who were equally competent with either both methods of subtraction (n = 11) or both methods of multiplication (n = 8) were selected.

Each student received a page with 20 lines composed of a random assortment of all 26 letters. Information processing was measured by the ability to accurately cross-out three specific letters among the random field of all letters. The instrument was administered before and after a 20-minute achievement test composed of either subtraction or multiplication problems. On the first day, half of the students worked the problems with the conventional method of computation and the other half with the Vedic Method. The method was reversed on the second day.

The speed and accuracy of information processing was determined by the number of

Table 4
Teacher Attitudes and Affect toward
Vedic Sūtra Based Computation—Pilot Study Number 3

Item	Percent strongly agreeing or agreeing (n = 43)
• I would like to learn more Vedic Mathematics.	92
• As a result of taking this course, I like the whole field of mathematics more.	85
• Learning Vedic Mathematics was fulfilling to me as a teacher.	92
• As a result of taking this course, I find more patterns and relationships among numbers.	85
• I am enthusiastic about Vedic Mathematics.	100
• Vedic Mathematics has taught me that there is more than one way to do a problem.	96
• I like Vedic Math more than regular math.	77
• I experienced feelings of bliss or great happiness often or occasionally while learning Vedic Mathematics.	100

letters canceled correctly minus the number of letters missed or canceled incorrectly. Evaluation of the change scores from before the computation test to immediately after, indicated that Vedic sutra based subtraction gave rise to significantly improved information processing ability as compared to conventional subtraction ($F(1,9) = 5.14$, $p = .048$, two-tailed), while the interaction of the type of method with the order in which the methods were used, was not significant ($F(1,9) = .204$, $p = .664$, two-tailed). In the case of multiplication, there were no significant differences between the methods ($F(1,6) = 1.538$, $p = .261$, two-tailed) and the interaction was not significant ($F(1,6) = .077$, $p = .779$, two-tailed). Analysis of covariance with repeated measures was used to evaluate the change scores. The pretest was the covariate.

Pilot Study Number Three

The third pilot study was an exploratory survey of attitudes and perceptions about Vedic sutra based computation among teachers. The subjects were 27 elementary and middle school teachers from Maharishi Schools in the United States, Canada, England, Norway, Kenya, and several other countries. The teachers had just completed an intensive one-week faculty development seminar in Vedic sutra based computation. A self-report survey given once at the end of the seminar showed that 92 percent agreed with the statement "learning Vedic Mathematics was fulfilling to me as a teacher"; 100 percent said they were "enthusiastic about Vedic Mathematics"; and 100 percent agreed they "often" or "occasionally" experienced "feelings of bliss or great happiness while learning Vedic Mathematics" (see Table 4).

Two essay questions at the end of the survey asked the teachers "What do you like most about Vedic sutra based computation?" and "What do you feel is its most valuable aspect?" The most frequent replies to the first question were "enjoyable" or "satisfying"; other responses (15) included: blissful, exciting, playful, satisfying, fulfilling, fun, and stimulating. The second most appealing aspect was its "ease" including 10 responses of ease, lack of strain, simplicity, little writing, and elegant. Other advantageous characteristics of Vedic Mathematics were also mentioned: speed (4 responses), enlightening (3 responses), and multiple ways to work a problem (2 responses). Concerning the second essay question, the two most valuable aspects of Vedic sutra based computation cited were that it was blissful, delightful, or motivating to the students (6 responses) and that it gave many choices to accommodate individual learning differences (6 responses). Other valuable qualities of Vedic sutra based computation were that it is easy and charming to teach and learn (5 responses), it develops consciousness or enlivens wholeness (3 responses), and it is logical and orderly (2 responses).

In summary, the literature review and pilot studies tend to indicate that many of the outcomes predicted by Maharishi do improve. Williams (1991) and the first pilot study found that Vedic Mathematics promotes more mental mathematics, an indication that computation does become more consciousness oriented. In the area of affect, Puri (1986), Williams (1991), and the first and third pilot studies found that practitioners of Vedic Mathematics experienced increasing ease and bliss, confidence, and motivation. In the area of performance, Tirtha (1965) and Puri (1991) state that Vedic Mathematics is easier and faster to use and learn. Based on these preliminary findings, an experimental study was designed.

An Experimental Study of Vedic Sūtra Based Multiplication and Checking

To test the claims that Vedic sutra based computation is fast, easy, enjoyable, and becomes more self-referral, and to evaluate achievement levels compared to conventional methods, the following four research hypotheses were constructed. Compared to students learning the conventional approaches for multiplying and checking at the elementary level, the following was predicted:

<u>Computational Skill</u>

Hypothesis one: Students learning Vedic Mathematics will exhibit (a) greater multiplication skill on tests of two digits multiplied by one digit, three digits multiplied by one digit, and two digits multiplied by two digits and (b) greater retention of that skill.

Hypothesis two: Students learning the Vedic sutra based method of checking will demonstrate greater checking skill.

Affective

Hypothesis three: Students learning the Vedic Methods will enjoy multiplication and checking more.

Mental mathematics

Hypothesis four: Students using *Ūrdhva* multiplication will perform more mental mathematics.

Subjects and Setting

The subjects were two classes of third grade students at Maharishi School of the Age of Enlightenment (Maharishi School) on the campus of Maharishi University of Management in Fairfield, Iowa. Maharishi School is a private, nonsectarian school, has a liberal admissions policy, and is accredited by the Iowa Department of Education and the Independent Schools Association of the Central States.

Subjects were randomly assigned by quartile based on their computation scores from the Iowa Test of Basic Skills to either the experimental or control group. Based on the pretest for Phase I ($t = .959$, $p = .35$), and a test of multiplication facts ($t = .416$, $p = .682$) given before the intervention began, the two groups were equivalent in multiplication skill.

Overview of the Curricula

The curriculum intervention began after the general concept of multiplication was uniformly introduced to both groups using manipulatives. At the third grade level, all students practice the Word of Wisdom technique to further develop their mental potential. Therefore, the curricula for both groups included practice of the Word of Wisdom technique for a few minutes at the beginning and end of the school day. Both groups shared the same mathematics teacher.

The conventional method of multiplication in this study refers to the abridged form of distributive multiplication (see Tables 5 and 6) traditionally taught in today's schools (Grossnickle, Reckzeh, Perry, & Ganoe, 1983, p. 165). The students learning conventional multiplication were taught to check their answers by either reworking the problem mentally or on paper (c.f., Grossnickle, Reckzeh, Perry, & Ganoe, 1983, p. 166). The teacher had five years of experience teaching this method at the third grade level.

The Vedic Mathematics curriculum intervention consisted of two Vedic Sūtra based algorithms. They were the *Ūrdhva* method of multiplication using the Vedic *Sūtra Ūrdhva-tiryagbhyām* and Vedic Check using the *Sūtra Gunita-samuchchayah samuchchaya-gunitah*. The teacher became qualified to teach Vedic Sūtra based computation by successfully completing a faculty development seminar on the topic given approximately seven months before the study. The following is an explanation of the Vedic Sūtra based methods used in this study.

Table 5
Comparison of Vedic and Conventional Algorithms
for Two and Three Digits Multiplied by One Digit

***Ūrdhva* Multiplication**	**Conventional Multiplication**
As taught in class:	As taught in class:
1) Multiply the one's place digit of the upper number by the single-digit multiplier:	1) Multiply the one's place digit of the upper number by the single-digit multiplier:
3 4 1 x 4 4	3 4 1 x 4 4
2) Shift the multiplier to the left one space and multiply vertically. This may be done by erasing it and rewriting, crossing it out and rewriting, or moving it mentally:	2) Multiply the ten's place digit of the upper number by the multiplier.
3 4 1 x 4 6/ 4 1	3 4 1 x 4 1 6 4
3) To find the hundreds place of the answer, shift the multiplier to the left another space and multiply vertically:	3) Multiply the hundred's place digit of the upper number by the multiplier.
3 4 1 x 4 1,3 /6/ 4 1	3 4 1 x 4 1 1,3 6 4

Table 6
Comparisons of Vedic and Conventional Algorithms for Two Digits Multiplied by Two Digits

Ūrdhva Multiplication

```
          16
         +21
          37
     8 7
   x 3 2
 2,7/8/4
   3  1
```

As taught in class:
1. Multiply vertically in the ones column.
2. Multiply crosswise and write the sum of the two answers.
3. Multiply vertically in the tens column.

Conventional Multiplication

```
     8 7
   x 3 2
      1
   1 7 4
    2
 + 2 6 1 0
   2,7 8 4
```

As taught in class:
1. Multiply ones by ones.
2. Multiply ones by tens.
3. Skip a space. Write a zero.
4. Multiply tens by ones.
5. Multiply tens by tens.
6. Add the two numbers.

Vedic Check

```
     8 7          6
 x   3 2      3   =   3
 2,7/8/4          5
```

As taught in class:
1. Add across to a digit-sum in each row.
2. Multiply the digit-sums of the two products. Reduce to a digit-sum.
3. If the digit-sum of the products equals the digit-sum of the answer, the answer checks.

Conventional Check

```
     8 7
   x 3 2
      1
   1 7 4
   2
 +2 6 1 0
  2,7 8 4
```

As taught in class:
Either rework the problem in your head checking to see if each step was done correctly or rework the problem on paper writing out each step.

Comparison of the Algorithms

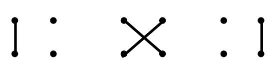
Three types of multiplication were taught in the intervention: two digits multiplied by one digit, three digits multiplied by one digit, and two digits multiplied by two digits.

One-digit multipliers. The general method[7] of Vedic multiplication is governed by the *Sūtra Ūrdhva-tiryagbhyām*:

ऊर्ध्वतिर्यग्भ्याम्
Ūrdhva-tiryagbhyām
Vertically and crosswise.
(Tirtha, 1965, pp. 40–50)

The first part of the meaning, "vertically," is used in the case of a one-digit multiplier (Puri, 1990, pp. 13–18; 1986, p. 63). The procedure is outlined in Table 5. In this approach, the one-digit multiplier is always directly beneath the digit to be multiplied, and the space for recording the answer is always directly beneath the multiplier. Thus, a strong vertical structure of multiplicand, multiplier, and answer is maintained during the steps of multiplication. This integration may make Vedic sutra based computation more coherent than the conventional method. The procedure for conventional multiplication is also outlined in Table 5.

Two-digit multipliers. The general method of Vedic multiplication for two digits multiplied by two digits is also governed by the *Sūtra Ūrdhva-tiryagbhyām* (Tirtha, 1965, pp. 40–51; Puri, 1986, p. 65). This *Sūtra* is often referred to simply as "*Ūrdhva*." The steps required by the *Ūrdhva* method are graphically represented below. Each dot represents a digit in the two factors shown in Table 6. Multiplication in the first step is vertical, in the second crosswise, and in the third vertical:



Step 3 Step 2 Step 1

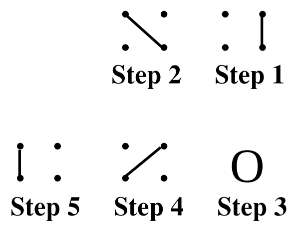
Notice that the graphic representation is symmetrical about its midpoint. This is a consistent characteristic of the *Ūrdhva* method no matter how many digits are multiplied. The symmetrical or balanced nature of the algorithm may increase its aesthetic appeal. Expert problem solvers note that aesthetics are important in problem solving (Silver & Metzger, 1989; Poincare, 1913). A more aesthetic approach is more charming or attractive to a problem solver and is, therefore, generally favored over an approach that is less aesthetically pleasing. This feature may make Vedic sutra based computation easier for children to learn.

Since children in this study were learning multi-digit multiplication for the first time, they were encouraged to write out the intermediate steps of multiplication and addition

[7]Vedic Mathematics provides a "general case" method for computation as well as "special case" methods (Tirtha, 1965, p. xx). The general case method may be applied to all types of problems within an operation such as multiplication. Special case methods may only be applied to specific types of problems within an operation. An example of a special case method within multiplication exists when both factors are near a base of 10 (i.e., a power of 10) as shown in Table 1.

around the parent problem as shown in Table 6. However, if so desired, students were allowed to do the work mentally. This flexibility of the algorithm accommodates different skill levels, those students who would rather do most of their work mentally as well as those students who would rather have a written record.

The written steps required by the conventional algorithm are graphically represented below. Each dot represents a digit in the two factors of the problem shown in Table 6:



Notice that the graphic representation of the conventional algorithm does not have the perfect symmetry characteristic of the *Ūrdhva* method. When compared to the symmetrical pattern of the *Ūrdhva* method, these steps appear broken, that is, the pieces of the pattern are symmetrical to each other but the pattern as a whole is not. This broken symmetry may hinder the aesthetic appeal or enjoyment of the conventional algorithm. Also, this format requires the student to write-out the intermediate steps allowing less opportunity for mental mathematics.

Conventional multiplication, as defined in this study, is a shortened form of the unabridged distributive algorithm. The advantage of this approach is that there is a written record of all the steps of multiplication and addition for the student or teacher to follow. Also, because the format requires the student to form two distinct rows of numbers to be summed, addition appears as a prominent part of the multiplication process, an attractive characteristic to some students.

Checking methods. The *Sūtra* governing Vedic Check is *Gunita-samuchchayah samuchchaya-gunitah*:

> गुणितसमुच्चयः समुच्चयगुणितः
> *Gunita-samuchchayaḥ samuchchaya-gunitaḥ*
> *The product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product.*
> (Tirtha, 1965, p. 89)

This *Sūtra* explains the Vedic checking process for multiplication and division (Tirtha, 1965; Puri, 1988b, 1986) as well as for addition and subtraction (Puri, 1986). The algorithm itself is mentioned in other sources as "casting out nines" (Cajori, 1928; Sovchik, 1977; Johnson, 1978; Stoddard, 1962; Grossnickle, Reckzeh, Perry, and Ganoe, 1983), but the *Sūtra* and its relationship to pure consciousness are not mentioned, an indication that its origin in the Veda is not recognized.

In Vedic Check, the student begins by comparing the digit-sums of the factors with the digit-sum of the answer. A digit-sum is the result of summing the individual digits of a row across to form a one-digit answer. For example, the digits in the answer from Table 6 are 2, 7, 8, and 4. The digit-sum for the answer is arrived at in the following way:

2+7+8+4=21, 2+1=3

Generally, a digit-sum may be found more easily by first "casting-out" the nines and any digits that add up to nine within each row:

~~2~~ ~~7~~ 8 4, 8+4 =12, 1+2=3

Thus, the process of finding the digit-sum for 2,784 is reduced to adding 8 and 4. Vedic Check follows the steps outlined in Table 6.

An advantage of using Vedic Check is that the student introduces a new Vedic sutra based algorithm and, therefore, the procedure for checking is completely different from the procedure for multiplying. This should decrease the chances of making the same procedural or calculation errors again during the checking process. However, by introducing a new algorithm, a new set of error patterns are also introduced. In the case of Vedic Check, there is a slight chance it will indicate that the answer is right even when it is wrong. This will happen when the answer is off by nine or a multiple of nine, if there are too many or too few zeros, when the decimal point is in the wrong place, or when digits in the answer are reversed (recorded in the wrong place value).

There are many conventional approaches to checking multiplication: reworking the problem mentally, reworking the problem on paper, interchanging the factors and multiply again, using a calculator to rework the problem, estimating the answer using upper and lower limits with rounding, or dividing the answer by one of the factors (c.f., Grossnickle, Reckzeh, Perry, & Ganoe, 1983, p. 166). Of these, two were selected for this study because they seemed most appropriate to the introductory level of multiplication: reworking the problem mentally or on paper. If students mentioned it, they were given the option of interchanging the factors, but this rarely occurred. Calculators were not used in the third grade, and the children had not learned how to divide.

Instruments Used to Evaluate Hypotheses

Because the two multiplication algorithms had special regrouping characteristics and each level of multiplication was to be evaluated separately, special multiplication achievement tests had to be created. Each problem on each test was screened by several professional educators. To evaluate checking skill, students used their respective checking techniques to find incorrect answers on their own work.

In the area of affect, the intention was to evaluate only enjoyment of multiplication, so special surveys were created. The items were adapted from the 1986 National Assessment of Educational Progress (Dossey, Mullis, Lindquist, & Chambers, 1988) and two pilot surveys. The checking affect surveys were created by adapting items from the multiplication affect survey and the survey administered in pilot study number one.

Structured, videotaped interviews with each student were carried out at the end of the study to determine student preferences. By this time, both groups had learned both meth-

Figure 1
Design of Maharishi's Vedic Mathematics Curriculum Intervention

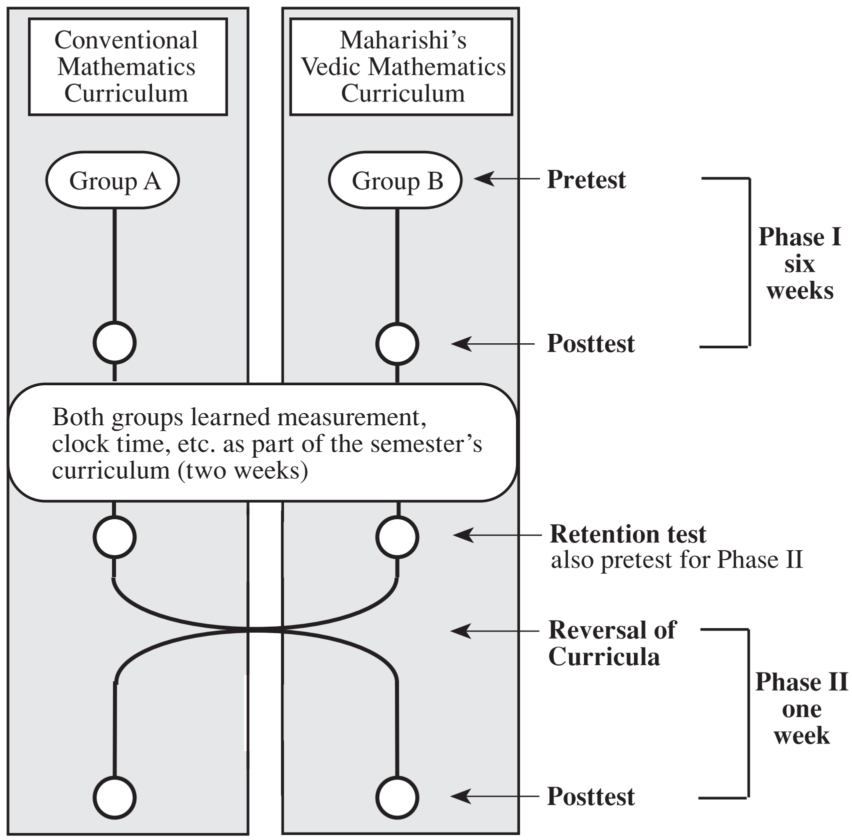


Figure 1. The study followed a pretest-posttest control group design that was carried out in two phases. Phase I lasted six weeks followed by a two-week break. Then, at the beginning of Phase II, the curricula were reversed and instruction continued for one week.

ods of multiplication and checking. The Strategy and Curriculum Assessment System (SCAS) provided a general measure of positive, negative, and neutral affective behaviors as well as attentiveness to the lesson. SCAS has been described as a system developed to code student or teacher behavior so that the final record preserves the original sequence of events as well as the duration of time of each event (Matthews & Matthews, 1990; Matthews, 1968).[8]

The average number of written digits needed to arrive at an answer was used to evaluate the amount of mental mathematics used in the conventional and Vedic algorithms. The evaluation of the posttest was identified as the most important result of the study since it comprised all levels of multiplication taught over the six weeks of Phase I, and because it represented the point when multiplication skill should have been maximum.

Design

The design of the study was a pretest-posttest control group design (Gay, 1987, p. 286) carried out in two phases as shown in Figure 1. Phase I of the design lasted for six weeks. During Phase I, the control group learned conventional multiplication and check and the experimental group learned the *Ūrdhva* method of multiplication and Vedic Check. This was followed by a two-week break when neither group was given instruction in multiplication. Phase II of the design followed the break and lasted one week. During Phase II, the curricula were reversed. Those who learned the conventional approaches to multiplication and checking during Phase I learned *Ūrdhva* multiplication and Vedic Check, and those who learned Vedic sutra based computation in Phase I, learned the conventional approaches to multiplication and checking.

Discussion of Results from the Study

Summary. During Phase I, students who learned the Vedic Methods had significantly higher overall achievement scores on all the most significant measures, which included the posttests for Phase I and Phase II and the retention test. The Vedic Mathematics group also completed significantly more problems with the same level of accuracy on all seven measures. Students using Vedic Check were able to find significantly more of their wrong answers on all three measures of checking skill. In the area of affect, the Vedic Mathematics group reported significantly greater overall enjoyment of both multiplication and checking on both surveys. However, SCAS scoring did not show any significant differences. (This was probably due to the curricula not illiciting student's affective reactions to the material being learned.) In the area of mental mathematics, the Vedic Mathematics group performed more mental mathematics than the conventional mathematics group on all four measures. Also, following the two-week hiatus after Phase I, test scores showed that the Vedic Mathematics group retained significantly more of the multiplication skill learned in Phase I.

After the curricula were reversed in Phase II of the study, the Vedic Mathematics group demonstrated significantly higher overall achievement scores as well as ability to find more of their wrong answers with Vedic Check. The Vedic Mathematics group also exhibited significantly greater enjoyment on all the most significant measures. Furthermore, structured interviews held after all students had learned both methods indicated that a significantly greater number of students liked the Vedic Methods better and desired to learn

[8]For the procedure used to administer the various tests or protocol of the structured interview or for a discussion of how the design controlled for various threats to validity such as controlling for a new approach, see Muehlman, 1994.

more about it during the following school year. None of the students said that learning two distinctly different methods of multiplying was confusing. A more thorough discussion of the study's results by hypothesis follows.

Hypothesis One: Multiplication Skill

The first hypothesis predicted that students learning the *Ūrdhva* method would exhibit greater multiplication skill on tests of two and three digits multiplied by one digit and two digits multiplied by two digits. Multiplication skill was measured in four areas: the number correct, total number completed, accuracy, and retention.

Number correct. At the time of pretesting, two achievement tests indicated that the groups were similar in multiplication skill. During the first two weeks of the intervention, tests of two digits multiplied by one digit and three digits multiplied by one digit also indicated no significant differences between the groups regarding total number of problems solved correctly. Then, during the third and fourth weeks, the Vedic Mathematics group worked significantly more problems correctly on tests of two digits multiplied by two digits without regrouping ($F(1,32) = 4.25$, $p = .022$, one-tailed) and two digits multiplied by two digits with regrouping ($F(1,32) = 6.48$, $p = .008$, one-tailed). The reason the Vedic Mathematics group suddenly improved may be due to distinct differences in the algorithms.

The Vedic and conventional algorithms for one-digit multipliers are not as distinctly different as the algorithms for two digits multiplied by two digits (see Tables 5 and 6). For one-digit multipliers, there are only slight differences in procedure, format used to record the answer, and opportunities for mental mathematics. However, there are significant differences between the Vedic and conventional methods for multiplying two digits by two digits in all three areas of procedure, format, and mental mathematics. Furthermore, it appears that these differences tend to enhance the performance of students using the *Ūrdhva* method as compared to students using the conventional method.

During the fifth week, a test of two digits multiplied by two digits with regrouping indicated that the Vedic Mathematics group was able to work more problems correctly, but the difference in number solved correctly was not significant. This outcome may have been due to the conventional mathematics group beginning to "catch-up" to the Vedic Mathematics group after two weeks of practice at two-digit multiplication with regrouping. Interviews with the teacher and students as well as pilot research which I conducted prior to the intervention indicated that the Vedic Methods were both easier and faster to learn. When a lesson is first taught, the Vedic Mathematics group consistently progressed more rapidly and large differences were observed between the groups. Even though the Vedic Mathematics group would continue to be more advanced, both the teacher and I observed that with time, the gap between the groups would lessen.

The most comprehensive measure of overall multiplication skill was the posttest given during the final week of Phase I. This test was a composite of all three types of multiplication taught during Phase I, and it was three times as long as previous skill tests giving students the opportunity to work as many as 60 problems depending on their level of proficiency. On this key test, the Vedic Mathematics group worked more problems correctly than the conventional mathematics group ($F(1,34) = 4.05$, $p = .025$, one-tailed).

After the two-week break, the Vedic Mathematics group showed greater multiplication skill on the pretest for Phase II. This outcome is discussed below as the retention test. Then, after the curricula were reversed for a week, the posttest for Phase II was given. Again, the Vedic Mathematics group worked significantly more problems correctly than the conventional mathematics group on this test of two digits multiplied by two digits with regrouping ($F(1,30) = 9.84$, $p = .002$, one-tailed). The robust performance of the Vedic Mathematics group after the reversal of treatments gives further evidence that the *Ūrdhva* method improves multiplication skill more effectively than the conventional method.

Retention. After Phase I of the study, there was a two-week hiatus during which both multiplication curricula were suspended and the groups learned other categories of arithmetic. At the end of two weeks, a retention test was given without notice. Results indicated that the students who learned the *Ūrdhva* method during Phase I retained significantly more multiplication skill than those who learned the conventional method ($F(1,32) = 3.41$, $p = .035$, one-tailed).

Results from a quiz given one week before the end of Phase I indicate that these results may not be attributed to one group having a better understanding of its algorithm. Scores on the quiz indicate that there were no significant differences between the groups in their understanding of the steps of their respective algorithms or what those steps meant in terms of place value.

Better retention by the Vedic Mathematics group may be explained by the responses of students during the structured interviews at the end of Phase II. Four students stated that they found the *Ūrdhva* method to be "easier to remember" than the conventional method. As one student explained, "This way is easier to remember because it's shorter and you only have to do that and then that and that and that [drawing the vertically and crosswise graphic]." The ease of remembering the *Ūrdhva* method may be due to the unified, symmetrical pattern of the *Ūrdhva* algorithm described earlier in this section. Due to its appealing symmetry, the pattern and sequence of the *Ūrdhva* algorithm may be easier to retain than that of the conventional algorithm.

Total number completed. On all tests of multiplication skill, including both posttests and the retention test, the Vedic Mathematics group completed significantly more problems than the conventional mathematics group with the same level of accuracy (F-values ranged from 4.53 to 20.50 and corresponding p-values from .020 to .0001, one-tailed). Again, this result may be explained by the responses from the structured interviews. The main reasons for preferring the *Ūrdhva* method was that it is "quicker" or "faster" (17 responses), and "shorter" or "may be done on one line" (8 responses). One boy expressed the practical value of being able to compute quickly by means of the *Ūrdhva* method:

> One time my Mom wanted to go on a big trip and she didn't know if she had enough money to pay for the gas to get there and she asked me to do it and I only had five minutes before I had to go back to school and I hadn't put on my uniform yet. So I used the Vedic Math way because that way is a lot faster.

One reason the *Ūrdhva* method may be faster is that it appears to be "less confusing" (6 responses). As one student explained during her interview:

> This way [pointing to the conventional method] is so hard and it's so long. First you have to go down this way then you have to go across this way. Then you have to put down a zero.

> Then you have to go across this way then you have to come down this way. Then you have to add all this up and it just takes hours. I like this way better [pointing to the *Ūrdhva* method] because it's much more interesting. It's less confusing because you just have to go like that and like that and like that and you don't have to add anything up and it's less boring. So I like it better.

Another student emphasized the creativity of the *Ūrdhva* method in association with its speed and ease:

> I like this way [referring to the conventional method] because it is harder and I like to learn new things, but I like the other way [referring to the *Ūrdhva* method] because it is a thousand times faster and more creative.

Perhaps, as experts in the field (Puri, 1991; Nicholas, Williams, & Pickles, 1984; Williams, 1984) have noticed, the creative, flexible format of the *Ūrdhva* method keeps the mind more alert thereby facilitating rapid computation.

There were no significant differences between the two groups regarding accuracy, the number right divided by number completed. This result indicates that the two approaches to multiplication are equally accurate.

The reversal of treatments in Phase II of the study confirmed the outcomes found in Phase I. During Phase II of the study, those who had learned the conventional methods during Phase I learned the Vedic Methods, and those who had learned Vedic Methods in Phase I learned the conventional methods. As mentioned above, the posttest for Phase II showed that the Vedic Mathematics curriculum resulted in more problems solved correctly and more problems completed. This consistent trend, in spite of the reversal of treatments, further demonstrates that the *Ūrdhva* method more effectively improves multiplication skill than the conventional method. It also confirms that descriptive terms such as *easier, faster, less confusing,* and *easier to remember* actually define the performance characteristics of the *Ūrdhva* method.

Hypothesis Two: Checking Skill

The second hypothesis predicted that the students who learned Vedic Check would demonstrate greater checking skill than those who learned the conventional approach to checking. Checking skill was measured by the success rate—the percentage of wrong answers marked as wrong—and the error rate—the percentage of right answers that were marked as wrong. A major issue in checking is that the same errors made during the original calculation will likely occur again while checking one's own work (Driscoll, 1981). In order to include the possibility of this type of error and therefore reflect more of a "real world" situation, students checked their own work on all three checking tests.

On both checking tests during Phase I—on problems of two digits multiplied by two digits with regrouping ($F(1,30) = 8.19$, $p = .004$, one-tailed) and on the posttest ($F(1,31) = 5.89$, $p = .010$, one-tailed)—Vedic Check was significantly more successful at detecting wrong answers than the conventional method of checking. This same pattern was also observed in Phase II when Vedic Check clearly outperformed the conventional method ($F(1,25) = 5.24$, $p = .014$, one-tailed). The reemergence in Phase II of the same pattern observed in Phase I strengthens the conclusion that Vedic Check is a superior approach to finding wrong answers in multiplication. There were no significant differences between the two groups regarding the error rate—the percent of right answers circled as wrong. The results indicate that both approaches are equally susceptible to this particular type of checking error.

Vedic Check, based on the *Sūtra Gunita-samuchchayah samuchchaya-gunitah*, introduces a new procedure of calculation. So systematic procedural or calculation errors (Kilian, Cahill, Ryan, Sutherland, & Taccetta, 1980, pp. 23–24) made with the *Ūrdhva* method are not likely to reoccur while using the *Gunita-samuchchayah* method. Conversely, Vedic Check also introduces a new set of potential error patterns not present in the *Ūrdhva* algorithm. However, the new set of potential errors is very small compared to that associated with the conventional method of checking. One student said,

> When it [referring to Vedic Check] checks, there's more knowledge knowing that it checks. What if you're making the same mistake over and over again. Here [pointing to Vedic Check] sometimes you can goof up too. Like if you're nine off or something. It can do that. But it's more likely to show that it's wrong than this one [pointing to the conventional method of checking].

Another student added,

> I like the Vedic Math way of checking because in the other way you don't really know if it's right . . . and the chances of it being nine off when with the other way [referring to the conventional method] there could be up to infinity different ways that it could be off, makes the Vedic Math way so much better.

Students also reported that they had a better chance of finding errors in their multiplication facts with Vedic Check as compared to the conventional method of checking,

> I don't like this way [referring to the conventional method] because say I thought 6 x 7 is 46 and I did that on my problem and then I checked it, I'd make the same mistake again. But using Vedic Check, I'd have a better chance of getting it right.

It is interesting to note that several students reported that the conventional method was not really a form of checking. One said, "You can't really check it with the older [conventional] way. No one really knew how to check before learning Vedic." Another said, "You do check . . . but it's not checking, you're just doing the problem over again and it's boring."

When the students' own steps of checking confirm the accuracy of their work, the whole process of computation becomes complete and fulfilling. Thus, checking can increase one's feeling of achievement and self-confidence. Also, students are freed from relying upon the teacher to confirm the answer. This increases the student's feeling of self-sufficiency. Even if the answer does not check, checking provides an objective means of reflecting on one's mathematical activity, which increases the student's knowledge and control of mathematics (Garofalo, 1987, pp. 48–49). A sixth grade teacher at Maharishi School comments on the value of Vedic Check:

> With Vedic Mathematics, the students don't have to come to the teacher to check if their answer is right. The ways of checking are easy so that the students want to check their own work. And when the right number comes up, they pop out of their seats saying "I got it!" The student is rewarded immediately. That's self-referral mathematics. There's a real thrill there.

Hypothesis Three: Multiplication and Checking Affect

The third hypothesis predicted that students learning the Vedic Mathematics curriculum would report more positive affect toward multiplication and checking than those students learning the conventional mathematics curriculum. This hypothesis was measured through the multiplication affect survey, checking affect survey, structured interviews, and SCAS.

The Vedic Mathematics group showed significantly more positive affective responses toward multiplication and checking than the conventional mathematics group. This outcome was observed on three of the four multiplication affect surveys, all three checking affect surveys, from the preferences stated on the survey administered at the end of Phase II, and from the analysis of responses made during the structured interviews at the end of Phase II. However, there were no significant differences between the groups on SCAS during Phase I or II. The positive results during Phase I, and the reemergence of the same pattern once the curricula were reversed in Phase II, strengthens the conclusion that affect is enriched more by the Vedic Mathematics curriculum than by the conventional mathematics curriculum.

Multiplication affect. The pretest for Phase I indicates that the Vedic Mathematics group and the conventional mathematics group had similar affective responses to multiplication. However, early in the intervention, differences began to emerge. During the first week of the intervention, after both groups had learned two digits multiplied by one digit, the teacher noticed several differences between the two methods with regard to ease of teaching and learning:

> It amazed me actually. I felt so much happier with the Vedic Math way. That way of doing 1 x 2 problems [referring to multiplication of one digit by two digits] made better sense. The children liked it more and understood it more quickly . . . . I truly felt that the Vedic Math group was more charmed by what they were doing.

The teacher's disposition was positively affected by *Ūrdhva* multiplication which she found to be easier and more charming to teach.

Notice that this teacher described *Ūrdhva* multiplication as making more sense and being easier to learn than conventional multiplication. Her experience may be due to the integrated, aesthetically pleasing nature of the *Ūrdhva* algorithm which may make it more charming. Even though the teacher believed that the Vedic Mathematics group liked what they were learning more, the self-report survey filled out by the students at the end of the first week did not show a significant difference between the two groups with respect to overall enjoyment.

Significant affective differences between the two groups were first measured when the students learned how to multiply two digits by two digits with regrouping ($F(1,29) = 5.66$, $p = .011$, one-tailed). At this point in the curricula, the two multiplication algorithms became much different: the abridged conventional method required two lines of multiplication and a third line for addition, whereas the *Ūrdhva* method retained its unified, one-line format.

Compared to the previous skill test, the Vedic Mathematics group improved 7.5 percent in enjoyment of multiplication (from a group mean score of 2.08 to 1.93) while the conventional mathematics group showed a 24.5 percent decrease in enjoyment (from a group mean score of 2.07 to 2.58). This difference indicates that the conventional mathematics group tended to lose interest in multiplication when their algorithm became increasingly fragmented and complex, while the Vedic Mathematics group retained its previous level of enjoyment. Perhaps the continued enjoyment of the Vedic Mathematics group is the result of the *Ūrdhva* method retaining its flexible format which, in the face of greater complexity, accommodated the learning style of the student. Since this change in enjoyment coincided with a major change in the format and complexity of the algorithms, the relative difference between the groups in overall enjoyment may be attributed to real differences in format and how well the algorithms handle complexity. The Vedic Mathematics group also reported significantly more enjoyment of multiplication during the posttest for Phase I ($F(1,34) = 6.08$, $p = .009$) and the posttest for Phase II ($F(1,30) = 31.39$, $p = <.0001$).

Results from both the structured interviews (chi square = 9.42, $p = .001$, one-tailed) and the survey given at the end of Phase II (chi square = 6.22, $p = .006$, one-tailed) indicated that the students preferred the *Ūrdhva* method over the conventional method, a result that was irrespective of which they learned first or spent most time mastering. The interviews indicated that students preferred the Vedic Method more ($p = .001$) and desired to learn more about it in the following year ($p = .0002$).

The responses most given by students for preferring *Ūrdhva* multiplication over conventional multiplication were that it was "faster" and "easier." From the final interviews in Phase II, there were 35 responses of "faster" or "easier" compared to 36 responses in all other categories combined. One student said, "It's faster, easier, and you don't have to write so much, and . . . [pause] it's just easier."

Several other reasons for preferring *Ūrdhva* multiplication proved to be quite insightful. One student said he liked it better because it was "more creative." Another said, "It's quicker and not as complicated as this one [pointing to the conventional method] where you have to write it down here and not over here [pointing to the empty space in the second line of multiplication]." This last response focuses on the confusion that may arise from using a space holder in the second line of the conventional method. In general, students found the format of the conventional method to be more confusing (6 responses) than that of the *Ūrdhva* method.

Another student stated that *Ūrdhva* multiplication was "easier for me. It's faster. It's only on one line," and another said, "You don't have to deal with a lot of numbers." These responses focus on the concise, unified format of the *Ūrdhva* algorithm which readily shows how all the parts of multiplication are integrated to form a unified whole; all the steps appear on one line in a logical, well integrated sequence that the student can easily grasp.

Two students commented that they wanted to learn more about Vedic Mathematics next year because its speed helped them on exams. One said he would like to "learn more about this one [pointing to the *Ūrdhva* method] because you might have some test that you want to go real fast on, so I'd like to learn more about that one." Another major reason for preferring Vedic sutra based computation was that students found it more interesting and so wanted to know more about it (5 responses). Several students (3 responses) also described its practical use in every-day situations as their main reason for wanting to know more about it next year. One student said:

> If you were at the store and your mom asked you to figure something out that needed to be multiplied, I'd use this one [pointing to the *Ūrdhva* method]. It's quicker. She wouldn't want to wait for about five minutes for me to finish the problem with this one [pointing to the con-

> ventional method] instead of waiting for only about a minute with this [pointing to the *Ūrdhva* method].

One of the most compelling responses indicating a preference for the Vedic Methods came during an open class discussion. After all the students had learned both methods of multiplication, they were asked to stand and say which they liked best and why. One student who had learned the conventional approach in Phase I asked, "Why did you teach us the regular way when you knew about Vedic?" The teacher replied, "We didn't know which one was better." The student confidently replied, "Well, we do!" feeling sure he was speaking for his whole class. During this open discussion, three other students voiced *strong* dissatisfaction with the conventional method. One stood and emphatically stated:

> I [dislike] it [referring to the conventional method] so much and I'll tell you why. One reason is that it is so complicated. It has more steps. It takes so long. It's all boring. It's hard to remember. I want to banish it from history. I like the other way [referring to the *Ūrdhva* method] *so* much better.

The above classroom discussion indicates that the sequence of steps for the *Ūrdhva* method are easier than the conventional method to learn, follow, and remember. Perhaps this is because the *Ūrdhva Sūtra* allows the brain to operate more in accord with Natural Law. According to Maharishi, the *Sūtras* are "refined formulas" that "reset the brain" so that it begins to operate in accord with Natural Law as the student computes. By operating more in accord with Natural Law, the brain begins to function more according to the principle of least action (see previous discussion)—from the unbounded, cosmic level of pure consciousness while computing. When students have the experience of finding a solution with less activity (more efficiency), it's natural for them to say the Vedic Methods are easier to learn and follow. By functioning more from the level of pure consciousness while computing, the student's intelligence or wakefulness is enlivened. This may explain why Vedic sutra based computation is easier to remember than the conventional approach.

This description of the value of the *Sūtras* to computation and learning is consistent with the experience of the teacher of both groups during the study. The teacher made the following comment while teaching a new lesson in two digits multiplied by two digits:

> The first group [conventional mathematics group] got bogged down . . . . Several students said, "I don't get it." The children in the first group felt resistant and tired. There was much more confusion there.

By being less of a strain to learn and use, the Vedic Method allows the body to feel more relaxed and less energy is consumed. The student uses less effort and enjoys multiplication more.

A surprising outcome of the student interviews was a reason for preferring conventional multiplication over *Ūrdhva* multiplication. Several students said that they preferred it because it was "more challenging" or had more steps (7 responses). One student stated,

> I like this way [pointing to the conventional method] because it's more challenging. Because there are more steps and I sometimes forget to put the zero right here. Sometimes I accidentally multiply [instead of add] so you have to remember more on it. There may be a little more adding or something; I don't know. I like it more.

Another major reason students gave for preferring the conventional algorithm was that its format handled carries better (4 responses). Students said that the carries were not as big with the conventional algorithm and that it was easier to find out where they were. Perhaps this complaint against the *Ūrdhva* method would be eliminated if a complete Vedic sutra based curriculum were offered. In such a curriculum, the student would have additional approaches such as *Nikhilam* multiplication that greatly simplifies multiplication of large factors (see example in Table 1). Due to the introductory nature of third grade multiplication and the desire to focus on the general Vedic Method of multiplication, only *Ūrdhva* multiplication was offered in this intervention.

Four students stated they would like to learn about both the Vedic and conventional methods next year. Although their specific reasons varied widely, generally they expressed that both methods had advantages and disadvantages. One student commented, "I'd like to have both because I'd learn more things. And if I get stuck on my favorite one, I could always do it the other way."

These contrasting preferences are another indication that the cause of affective enrichment is due to real differences in the computational techniques and not to inequalities in instruction or built in biases. Student preferences were not superficial; rather they were thoughtful and consistent. Preferred choices confirmed perceptions held by experienced elementary and secondary teachers. Moreover, they were consistent with the experience of experts in the field such as Puri (1986) who has routinely applied the Vedic Methods in public education settings.

Checking affect. On all three checking affect surveys, students using Vedic Check indicated they experienced more overall enjoyment than students using the conventional method of checking (F-values ranged from 18.95 to 109.06 and corresponding p-values ranged from .002 to <.0001). The major reason for the greater success of Vedic Check was that students enjoyed it. Results from the structured interviews showed that 95 percent of the students (35 out of 37) preferred Vedic Check to the conventional means of checking. They consistently cited Vedic Check as being "much more interesting," "less boring," "uses less numbers," or "lots of fun." One student said,

> You can't keep your mind on what your checking [with the conventional method of checking]. In this one [pointing to Vedic Check], you don't have to deal with a lot of numbers. It's just easier.

Several students said they preferred Vedic Check more because they felt confident it proved their answers either right or wrong. As one student said regarding checking with the conventional method, "you never know if you got it right." The confidence and fascination students felt with Vedic Check caused them to want to use it. Results from the structured interviews at the end of Phase II confirmed the student preference for the Vedic Method (chi square = 28.42, p = <.0001, one-tailed).

Affective behavior. As noted above, the self-report measures and interviews indicated that the Vedic Methods enrich affect more than conventional methods. However, the six days of observation with SCAS during Phase I and two days of observation in Phase II, student behaviors during class showed no significant differences in positive, negative, or neutral affective behaviors. This lack of a predicted outcome is attributed to an unusually high degree of contentment at Maharishi School. Also, the curricula did not require the teacher to regularly illicit student's affective reactions about the material being learned.

So the SCAS scorer observed far fewer and subtler affective behavioral distinctions leading to a lack of conclusive results.

The teacher's responses during biweekly interviews addressed the issue of learning equality. During the first interview at the start of the intervention, the teacher said, "I feel equally excited about both curricula." At first, she felt more fatigue during the second period which actually worked against the Vedic Mathematics group. However, her fatigue diminished once she became more familiar and confident teaching the Vedic Methods.

Results confirmed by literature and pilot studies. Enriched affect was also reported in 1988–1989 by students who had been learning Vedic sutra based addition, subtraction, and multiplication for a full semester at Maharishi School. A pilot survey of ninety-three 3rd–5th graders indicated that 78 percent liked Vedic Mathematics (see pilot study number one). Furthermore, this outcome is consistent with observations of experts such as Puri (1986) and Williams (1991) who assert that Vedic sutra based computation reduces monotony and anxiety while increasing ease and enjoyment.

The affective enrichment of Vedic sutra based computation does not seem limited to specific cultures or age groups. Increased enjoyment and motivation have been reported by teachers in educational settings in the United States, England, India, Norway, Canada, and Kenya (see pilot study number three), among elementary and middle school students (see pilot study number one), and adults in continuing education programs (Puri, 1991). Also, increased happiness with Vedic sutra based computation is not limited to a few formulas; rather a wide range of different algorithms and *Sūtras* produce similar waves of enjoyment (Puri, 1986). The agreement of results from this study with other research and expert opinions tends to confirm that Vedic sutra based computation is more enjoyable and motivating than the conventional methods.

Hypothesis Four: Mental Mathematics

The average number of written digits used in the intermediate steps of multiplication was the first measure of mental mathematics. Even though the *Ūrdhva* algorithm does not require students to write out all the intermediate steps of multiplication and addition as does the conventional algorithm, since this was an introduction to the *Ūrdhva* method, students were actually encouraged to write out the intermediate steps of computation in small problems surrounding the parent problem. These intermediate steps were repeatedly demonstrated by the teacher from the board and in teaching materials. However, when students asked if they were required to write out the steps, they were told they could do as much or as little of the work in their head as they would like.

Digits in small problems and "scribbles" worked in the space surrounding the parent problem were counted as part of the intermediate steps. The amount of this kind of written material provided a general method for comparing how much mental computation a student used when working a problem. It was assumed that less written digits meant more mental computation took place. Since the students were not required to solve the problems on one line, and because they were encouraged to write out the intermediate steps, the number of digits in the intermediate steps indicate how much mental calculation was performed by the student.

All four evaluations document that the Vedic Mathematics group used significantly fewer written digits to arrive at an answer than the conventional mathematics group (F values ranged from 24.34 to 356.05 and corresponding p values were all <.0001), and suggests that the *Ūrdhva* algorithm cultures more mental computation than the conventional distributive algorithm.

A second evaluation counted the number of times students in the Vedic Mathematics group elected to work the "crosswise" step of the *Ūrdhva* method mentally. The crosswise step is considered to be the most demanding step of the algorithm to work mentally. The Vedic Mathematics group chose to work it mentally 54 percent of the time on the skill test of two digits multiplied by two digits without regrouping, and on the test of two digits multiplied by two digits with regrouping, 30 percent of the time. Together, these two evaluations indicate that the flexible format of the *Ūrdhva* method cultures the ability to perform mental computation. As students use the *Ūrdhva* method over time, they naturally tend to perform much more of their computation mentally.

These findings are supported by the personal interviews. Three students stated the *Ūrdhva* method could be done more easily "in the head" or "in my mind." One student said that he "would like to learn more about this one [pointing to the *Ūrdhva* method] because it's faster to do things in the head." These responses are consistent with those given by experts in the field of Vedic Mathematics such as Tirtha (1965) and Williams (1991). Tirtha points out that "the *Sūtras* are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarized in one word 'mental'!" (Tirtha, 1965, p. xvi).

Increased mental computation is not limited to the *Ūrdhva* or *Gunita-samuchchayah* methods. One year before this study, I introduced *Nikhilam* multiplication to a third grade class (see example in Table 1). Within about 10 minutes of learning the algorithm for the first time, several students were able to easily stand in class and speak out the solution to problems such as 98 x 97 without written help.

Trafton (1978) and Williams (1991) say that mental computation adds vitality to computation, increases flexibility, gives new insights into numbers and number relationships, and develops problem-solving skills. In light of these practical advantages, the finding that the *Ūrdhva* method increases mental computation is sufficient to recommend its use in mathematics education programs.

Extension of Results to Explain Possible Growth in the Direction of All Knowingness

While looking at the results related to the four hypotheses, it became apparent that when all the results were taken together, there may be indications of growth in the direction of mathematics without steps. This final section presents this possibility.

As one grows towards all knowingness or mathematics without steps, it is predicted that computation becomes faster, easier, and increasingly self-referral—based in consciousness. Results from this study indicate that (1) the Vedic Mathematics group was able to solve more problems correctly than the group using the conventional method with the same level of accuracy, (2) most students preferred the *Ūrdhva* method, found it to be faster and easier and to have less steps, and (3) the Vedic Mathematics group spontaneously performed more mental mathematics than the conventional mathematics group, an indication that computation was based more in consciousness. This combined evidence supports the proposition that the Vedic Mathematics group experienced growth toward mathematics without steps.

Increased mental mathematics may be explained somewhat by the flexible format of the *Ūrdhva* method which provides more opportunity for mental computation. However, the flexible format does not explain why students chose, or tended naturally, to do more mental computation. It may be that practice of the Word of Wisdom technique gives students access to deeper levels of consciousness, and with addition of the Vedic *Sūtras*, these deeper levels become enlivened, naturally organizing the mind to compute in accord with Natural Law. Individual intelligence becomes more closely aligned with cosmic intelligence and begins to compute through the principle of least action. This enlivenment of consciousness creates the ability to compute rapidly, precisely, and easily at deeper, more abstract levels of the mind. Evidence, such as the shift towards a more abstract, mental form of computation through the use of Vedic sutra based computation, tends to support this assumption.

If this developmental sequence were to be continued to its logical culmination, the student may develop mathematics without steps. By so doing, the student would be able to know anything spontaneously and solve all problems in any area of life automatically, effortlessly on the level of pure consciousness, without making mistakes. Here is a simple model of computation to explain the developmental process more clearly. In this model, three classifications of computation are shown—from gross, to subtle, to transcendent:

> Third class computation: At the most concrete level of mathematics, the steps of computation are worked out on paper or with a calculator requiring many steps.
>
> Second class computation: At a more refined level of mathematics, the steps of computation are easily worked mentally requiring fewer steps.
>
> First class computation: At the most developed level of mathematics, the solution to any problem comes spontaneously to the awareness allowing one to experience the bliss of all knowingness. This is mathematics without steps, the full awakening of the Cosmic Mathematician in human awareness.

We have seen that even young students using the *Ūrdhva* method tended to work large portions, or an entire problem, mentally. Based on the above model, this is a shift away from the concrete procedures and representations of third class computation towards more abstract mental procedures and representations characteristic of second class computation. It should also be noted that this shift occurred naturally, without any instructions or encouragement from the teacher. Also, with this shift toward more self-referral computation, results indicated that mental processing of mathematical information, affective enjoyment of calculating, and the percentage of problems worked correctly, all improved. That is, speed and ease of computation improved and the number of perceived steps became less. This indicates a holistic improvement in computation.

To summarize, the shift from third class computation to second class computation results in (1) more mental computation, (2) increased enjoyment, (3) solving more problems correctly, (4) fewer steps of computation, (5) less effort, and (6) less time needed to compute. These outcomes suggest that Vedic sutra based computation develops the student's awareness in the direction of all knowingness, the ability to think and act spontaneously in accord with Natural Law.

If these qualities acquired by performing Vedic Mathematics continue to grow, which may be the case since the benefits of other aspects of the Vedic Mathematics program such as the Word of Wisdom technique are cumulative, then it is reasonable to say that the developmental direction indicated by increased mental computation may shift to self-referral computation. As this shift takes place, it is predicted that the steps of computation become fewer and fewer, effort continues to decrease, bliss increases, the interval of time of computation becomes shorter, and the percentage of correct answers rises. This growth may culminate in a shift from the mental procedures and representations of second class computation to fully established first class computation, mathematics without steps, wherein the solution to any mathematical inquiry spontaneously comes to the awareness.[9]

It is interesting to note that so many diverse improvements may be attributed to one treatment. Results reported in this article indicate that improvements in mathematics performance, overall enjoyment, and mental mathematics may be attributed to Vedic sutra based computation in conjunction with the practice of the Word of Wisdom technique. Also, studies specifically on the Word of Wisdom technique demonstrate that it has a coherent or integrating effect on the functioning of the brain and its related cognitive activity. This data taken together, suggests that Maharishi's Vedic Mathematics program at the elementary level develops the student holistically.

The main premise of this article has been that, in the context of a full Maharishi's Vedic Mathematics program, Vedic sutra based computation cultures the ability of the student's mind to function from the level of self-referral consciousness—the Unified Field of all the Laws of Nature—while computing. This gradually develops the ability to think and act in perfect accord with all the Laws of Nature. The student begins to compute more in accord with the principle of least action which makes computation easier, faster, more mental, and more enjoyable. Over time, it is predicted that mathematics without steps—the state of all knowingness—will develop in the student's experience. This theme has practical implications relevant to modern mathematics education: Jeremy Kilpatrick states that there is a need for more "self-awareness," the ability of students or teachers to be "more conscious of what they are doing when they learn or teach" mathematics (Kilpatrick, 1985, p. 1). Also, metacognition—the knowledge of how the mind thinks, the awareness of how thinking takes place from moment to moment, and gaining control over the thought processes—has become a growing concern among mathematics educators interested in

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[9]Historic records document individuals who have displayed a few characteristics of mathematics without steps: instantaneous, effortless, and accurate computation. It should be noted that the following are examples of a specific skill and not the result of a systematic program for the holistic development of consciousness.

During a special meeting of the Institute of Civil Engineers, Professor Bidder (1856) said that he could multiply two numbers together "in what appears to be merely an instant of time; and I can do any quantity of the same sort of calculation without any labor; and I can continue it for a long period." In 1954, Professor Aitken described his experience of instantaneous calculations to the Society of Engineers: "I have noticed at times that the mind has anticipated the will; I have had an answer before I even wished to do the calculation; I have checked it, and am surprised that it is correct." Even though a few characteristics of mathematics without steps are exemplified here, there is no implicit assertion that the level of consciousness lived by these individuals is the same as that of mathematics without steps.

improving problem solving (Kilpatrick, 1985; Schoenfeld, 1983; Silver, 1982). Moreover, George Polya cites a central need of mathematics education when he asks, "What can the mathematics teacher do in order that his teaching improves the mind?" (Polya, 1983, p. 1). One answer is Vedic sutra based computation taught in the context of a full Maharishi's Vedic Mathematics program. This approach to computation not only proves to be more enjoyable and motivating than conventional methods for both students and educators, it is a system that also develops the student's consciousness while learning mathematics. Maharishi's Vedic Mathematics creates mathematicians with enlightened minds—people who steadily grow in the ability to spontaneously know anything and achieve anything with only beneficial effects for all of humanity.

Appendix

Review of Additional Characteristics of a Full
Vedic sutra based Computation Program

Multiple Choice of Methods

In Vedic Mathematics, Puri explains that the first step is seeing the pattern the solution will take and then selecting the appropriate *Sūtra* (Puri, 1991, p. 17). In deciding the best pattern, students may choose among the various approaches. Vedic Mathematics provides a "general case" approach for computation as well as "special cases" (Tirtha, 1965, p. xx). The general case approach may be applied to all types of problems within a single operation. An example is multiplying with *Ūrdhva-tiryagbhyām Sūtra* which may be used to multiply in all cases (see Table 1). However, special case methods may only be applied to specific categories within an operation. Multiplying two numbers that are near a base of 10 using *Nikhilam Sūtra* is a specialized method of multiplication (see Table 1). Having both general and special case approaches gives the student greater freedom to select the path of least action and maximum enjoyment while computing.

As an example, the problem 87^2 may be solved with the *Ūrdhva* method, *Nikhilam* multiplication, or by applying a special case method based on the *Yāvad ūnaṁ tāvad ūnīkritya vargam cha yojayet Sūtra* (referred to as the *Yāvad ūnam Sūtra*) for squaring near a base of ten. Let's say *Nikhilam* multiplication is chosen. In the *Nikhilam* algorithm, the right-hand side of the answer is found by multiplying the deviations of the two factors from 100. The deviation of 87 from 100 is –13. Either the *Ūrdhva* or *Yāvad ūnam Sūtras* could be used to multiply –13 x –13 as shown:

	Parent Problem	Auxiliary Problem: –13 x –13		
	Nikhilam	*Yāvad ūnam*	or	*Ūrdhva*
Solve 87^2	87 –13	(13 + 3) /3^2		13
	x 87 –13	16 /9		x 13
	75/ 69	169		1/6/9
	1			

Then, returning to the parent problem and the *Nikhilam* algorithm, the left-hand side of the answer is found by cross-adding 87 + (–13) = 74. Then add the carry, 74 + 1 = 75.

This element of choice in the Vedic System "brings fun and amusement." Williams notes, "the mental mathematics leads to a more agile, alert and intelligent mind, and innovation naturally follows" (Williams, 1984, p. vi). Professors Nicholas, Williams, and Pickles describe the flexibility of this method as "the great benefit of the approach, particularly when taught to younger students. Presented by a skillful teacher, its simplicity and ease readily shine forth" (Nicholas, Williams, & Pickles, 1984, p. viii).

Puri (1991) observes that children love to play. When they are asked to do something in the same way over and over again, in a rigid pattern, boredom and anxiety set in (Puri, 1991, p. 12). In Vedic sutra based computation, the operations are highly flexible. The student has several choices of how to solve any problem. Puri notes that this flexibility eliminates rigidity and increases the opportunity to play with numbers.

Flexibility of Notation

In Vedic Mathematics, numbers are often considered in relation to a base (Nicholas, Williams, & Pickles, 1984, p. viii). Thus 8 is seen as being 2 away from the base of 10, and 179 has a deficiency of –21 from the base of 200. The Vinculum method utilizes this concept to create an alternative system of number notation.

Vinculum[10] allows any number to be represented by using only digits 0 through 5, eliminating the need to compute with large digits (Tirtha, 1965; Puri, 1988a). For example, the number 179 may be converted to 221 through the Vinculum method. The converted number is known as a Bar number. It retains the computational properties of the original number but the digits are smaller and so easier to work with. The conversion to smaller digits is governed by the *Nikhilam Sūtra*, "all from nine and the last from ten":

> Example: convert 179 into a Bar number
>
> Apply Nikhilam *Sūtra* to the 7 and 9: nine from 10 is 1 and seven from 9 is 2.
> Add bars to these digits. Finally, increase the 1 in the hundreds place by 1.
> 1 7 9 = 2 2 1
> This Bar number is read "two, two bar, one bar."

In this example, bars indicate that the digits below them are negative. So if 221 is interpreted according to place value, it is really 200 – 20 – 1. Thus, Bar numbers are always composed of both positive and negative digits.

Nicholas, Williams, and Pickles (1984) point out that the elimination of big digits "lightens the task" of computing considerably in most cases. Puri also notes that "whenever we are operating with the bigger digits (6, 7, 8, 9), it requires slightly more effort and a little more time compared with the operation of smaller digits" (Puri, 1991, p. 12). The process for normalizing back to regular notation is also part of the Vinculum method. Once a student has mastered Vinculum, converting and normalizing a number occur mentally in as much time as it takes the student to write the number.

The combined flexibility of method, notation, and format indicate that Vedic sutra based computation is adaptable to a wide range of different learning styles and individual preferences. As Puri and Weinless points out, "The techniques of Vedic Mathematics allow for constant expression of a student's creativity" (Puri & Weinless, 1988, p. 2). The flexibility of method, notation, and format "keeps the mind lively and alert and cultures the ability to quickly discover the path of least action on the way to the solution."

Integration of Brain, Development of Intuition

As mentioned above, the first step of Vedic sutra based computation is seeing the pattern that the solution will take and selecting the *Sūtra* that corresponds to that pattern (Puri, 1991, p. 17). Puri (1991) notes that seeing the pattern of the problem

> essentially involves systematic use of the right half of the brain. Then the logical computations are done by the left half. Even while computing, due to the various options available, the mind is always kept alert to pick up the path of least effort. (p. 15)

Puri goes on to say that going back and forth, from pattern recognition to analysis, may develop the brain holistically. This flexibility of Vedic Mathematics at each stage of problem solving "keeps the mind lively and alert and develops clarity of mind and intuition" (Puri, 1989, p. 7).

One Set of Techniques for All of Mathematics

Tirtha explains that Vedic Mathematics provides an integrated approach to mathematics where all mathematical computation and relationships are an elaboration on the knowledge contained in the 16 basic *Sūtras* (Tirtha, 1965, p. xvi). Nicholas, Williams, and Pickles (1984) concur by pointing out that the operations of elementary level Vedic Mathematics prepare one for algebra.

In algebra, the polynomial acts as a generalization of positional notation (Nicholas, Williams, and Pickles, 1984, p. ix). Once this connection between arithmetic and algebra is understood, one may see how Vedic Mathematics provides an efficient system of computation that may be applied to both arithmetic and algebra. So once the student has "learned arithmetic methods, very little extra is required to learn algebraic ones" (Nicholas, Williams, and Pickles, 1984, p. ix). The *Ūrdhva* method of multiplication will illustrate.

Using the *Ūrdhva* method of multiplication exemplified in Table 6, the answer to 12 x 34 is **(3x1)/(4x1)+(3x2)/(4x2)** where the slashes (/) separate the place values. Notice how the slashes force the products to appear in certain place values thereby factoring out 10 and 100 from the middle and left parts of the multiplication respectively. This one line format is identical to the algebraic problem (ax+b)(cx+d) where a,b,c,d, are the digits 1,2,3,4 respectively, and x = 10. When the algebra is multiplied, it equals $acx^2 + x(ad+bc) + bd$ which may be written as **ac/ (ad+bc) / bd** if place value is used to factor out the "x" from the middle and left terms.

Another way elementary level Vedic Mathematics prepares a student for algebra is by using both positive and negative digits:

> The digits recorded in positional notation are, by custom, all positive, whereas the constants of a polynomial can be positive or negative. The Vedic scheme uses both positive and negative digits however, which deals with this point. (Nicholas, Williams, & Pickles, 1984, p. ix)

[10]Vinculum itself is not a Vedic *Sūtra* but a notational method which is governed by a Vedic *Sūtra*. References to Vinculum may be found in other sources (Cajori, 1928, p. 57; Ballantine, 1925, p. 302). However, the *Sūtra* that governs the method and the connection to pure consciousness are not mentioned.

Thus, the polynomial $x^2 - 9x - 3$ where $x = 10$ may be seen as an algebraic representation of the number 7: $x^2 - 9x - 3 = 100 - 90 - 3$ which equals 193 in Vinculum notation.

If all mathematical algorithms and relationships can be derived from the *Sūtras* of Vedic Mathematics, then mathematics can become simpler and easier to handle. Instead of an endless series of formulas and procedures, students will find that mathematics unfolds from the knowledge contained in just 16 Vedic *Sūtras*. For example, *Ūrdhva Sūtra*, used to multiply in the third grade, is still applicable throughout secondary and post secondary mathematics education to solve simultaneous linear equations, invert a matrix, or solve transcendental and differential equations (Nicholas, Williams, & Pickles, 1984).

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