Vedic Mathematical Concepts and Their Application to Unsolved Mathematical Problems: Three Proofs of Fermat's Last Theorem

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Preface

The following three proofs, based on the application of Vedic Mathematical concepts, address a famous unsolved problem of mathematics, Fermat's Last Theorem. The following passage of Jabali Upanishad provided the structural key for developing the multi-dimensional spaces used in the argument for Fermat's Last Theorem:

> अथ हैनं भगवन्तं जाबालिं पैप्पलादिः पप्रच्छ भगवन्मे ब्रूहि परमतत्त्वरहस्यम् ॥१॥ किं तत्त्वं को जीवः कः पशुः क ईशः को मोक्षोपाय ति ॥२॥ स तं होवाच साधु पृष्टं सर्वं निवेदयामि यथाज्ञातमिति ॥३॥ पुनः स तमुवाच कुतस्त्वया ज्ञातमिति ॥४॥ पुनः स तमुवाच षडाननादिति ॥५॥ पुनः स तमुवाच तेनाथ कुतो ज्ञातमिति ॥६॥ पुनः स तमुवाच तेनेशानादिति ॥७॥ पुनः स तमुवाच कथं तस्मात्तेन ज्ञातमिति ॥८॥ पुनः स तमुवाच तदुपासनादिति ॥९॥

> Then Paippaladi asked Lord Jabali, "Tell me, Lord, the secret, supreme reality. What is tattva [existence]? What is jiva [individual life]? What is pashu [the soul]? Who is Ish [the Master]? What are the means to enlightenment?" He said to him, "Very good! Everything that you have asked, I will explain to you, as it is known." Again he said to him, "How is it that you know this?" Again he said to him, "from Shadanan." Again he said to him, "How does he then know this?" Again he said to him, "from Ishan." Again he said to him, "How does he know it from him?" And again he said, "from upasana [worship]."

In an earlier work, *The Rationale of Om—Its Formulation, Significance and Occurrence and Applications in Ancient Literature,* I commented on the above passage as follows:

> The import in the above nine mantras of the Jabali Upanishad is that Paippaladi had asked Jabali Rishi to enlighten him about tattva, jiva, pashu and Ish. Jabali Rishi happily prepared to instruct him about the questions asked. Just as Jabali Rishi was about to begin his discourse, Paippaladi inquired how he Jabali Rishi had achieved enlightenment.

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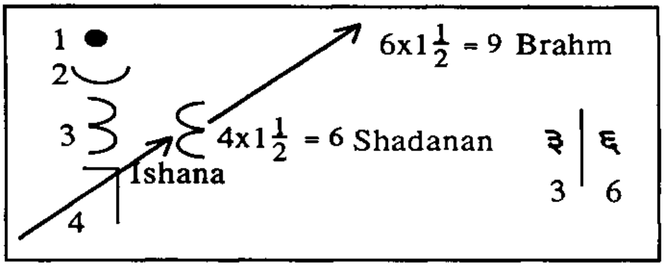
On this, the Jabali Rishi disclosed that he was enlightened on the points by Shadanan. On this he further asked from where Shadanan had achieved enlightenment and to this the answer of Jabali Rishi was that Shadanan had received enlightenment from Ishan.



Shadanan to Ishan

The insight of descendence of Brahmavidya for the Lower Mathematical domain is that the processing should begin from within the fourth-dimensional domain in terms of its constituent, that is a plane is to be along its diagonal which would be facing North-East (Ishan) and would be leading to the sixth-dimensional domain (Shadanan). Sequential order would emerge because of dedh-devata (डेढ देवता). The processing which was initiated from within the fourth-dimensional domain led to the sixth-dimensional domain. The processing was along the diagonal. The diagonal is greater than either side. The diagonal is also less than the sum of both the sides (of the triangle). This concept is the concept of one and a half units. The processing line of one and a half units is 4x3/2=6. The Upanishadic enlightenment on the point is that the devas are 1, 3/2, 2, 3, and so on (Brihadaranyaka Upanishad). The concept of dedh-devata (डेढ देवता) is the specific processing concept. In terms of this concept the processing along the North-East line (Ishan) which has taken the fourth-dimensional domain to the sixth-dimensional domain sequentially would carry the processing further, naturally to the ninth-dimensional domain as 6x3/2 = 9.

The significance of the above within the arithmetic domain is that in order to understand the structural frames and systems of natural number 9 (in Upanishadic language: Brahmavidya) we have to develop the understanding in terms of the structural frames and systems of natural number 6 (in Upanishadic language Shadanan bestowed enlightenment upon Jabali Rishi). And understanding the structural frames and systems of natural number 6 must be in terms of the structural frames and systems of natural number 4 (in the Upanishadic language Ishan bestowed enlightenment upon Shadanan). This reverse sequential process contains the structural key.

The basic Vedic Mathematical concepts used are that the unity (single-syllable Om) is processable quarter by quarter **(Shri Pada processing line)** and the fourth quarter is the integration of the first three quarters **(Maharishi processing line).** As such, I interpret the above Upanishadic passage in the following way: the structural frames and systems of natural number 4 (and hence the fourth-dimensional domain) are to be handled as unity, admitting processing quarter by quarter, and the fourth quarter (the unmanifested quarter) is to be processed as integrating the first three quarters (manifested quarters). The processing within the fourth quarter along the North-East diagonal would lead to the domain of Shadanan (admitting structural frames and systems of natural number 6 and hence a six-dimensional domain). The Upanishadic command is to reverse the process, as the processing is to be had in the descending order from 9 to 6 and 6 to 4, but here in this world everything is hanging upside down (ऊर्ध्वमूलम् urdhvamulam—Bhagavad-Gita, 15.1). This means that while our aim is to process within the unmanifested quarter in order to go from the fourth-dimensional domain of natural number 4 to the sixth-dimensional domain of natural number 6, we have to process along the North-East diagonal but in the direction which leads from the sixth-dimensional domain towards the fourth-dimensional domain. Further, the processing must be quarter by quarter, which means that when natural number 4 is taken as a unity, its quarter would be one, and similarly, when natural number 6 is taken as a unity, its processing unit would be one. Hence the processing along the North-East diagonal is to be had by dividing it into six parts. Out of these six parts, only one part would be unmanifested and the remaining five parts would cover the manifested part. With this the procedure of structural processing stands as follows:

Step 1: The processing is to be in the fourth quarter of the Om formulation.

Step 2: Fourth quarter (fourth component) of the Devanagari Om formulation is like a two-dimensional cartesian frame.

Step 3: So processing within the fourth quarter (fourth component) amounts to processing on the format of a plane.

Step 4: As both the dimensional lines are symmetric, the natural figure of the plane format is a square (to be called the processing square).

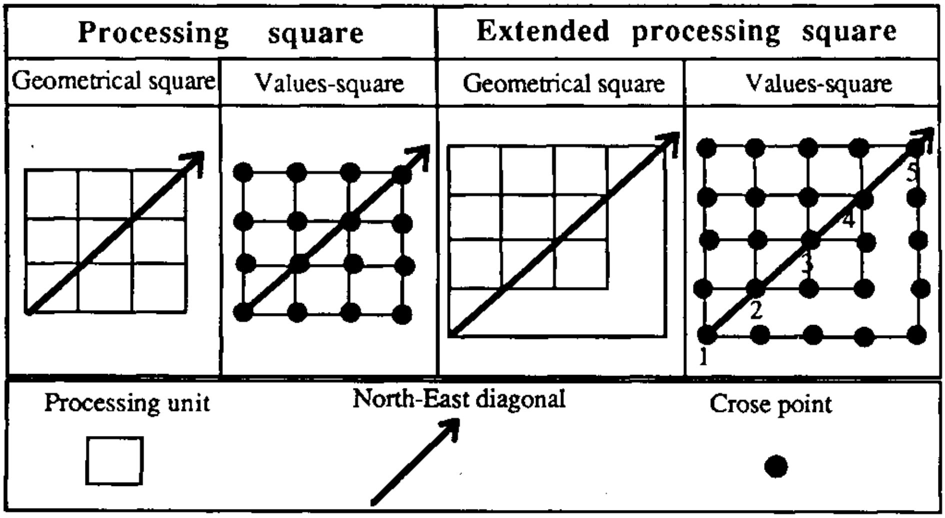
Step 5: For processing of the first three quarters of the manifested domains, divide the square into $3 \times 3 = 9$ squares (to be called processing units).

Step 6: Therefore, if the length of the square is Z, then the length of the processing unit would be $Z/3$.

Step 7: As the fourth quarter is to be processed as the integration of the first three quarters, the length of the processing square should be increased by the length of its processing units, that is, the extended processing square, as it may be called, would have a length equal to $Z + Z/3 = 4Z/3$.

Step 8: The processing square being of length Z, the manifested part of the North-East diagonal of the extended processing square would be $5Z/6$, and the unmanifested part of the North-East diagonal of the said square would be $Z/6$.

Step 9: The above format develops a square whose length is equal to the diagonal, which means we are transcending from the geometrical square to the values-square (as it may be called). In this format transcendence is permissible within the format of a geometrical square itself in terms of the cross points of the lines parallel to the axes of the two-dimensional cartesian frame of the geometrical square. If we take into account only the cross points of the lines parallel to the axes (which lines may be called parallel axes) then the cross points on the length or breadth of the square would be equal to the cross points on any of the two diagonals of the square.



Step 10: The effect of extending the processing square into the extended processing square upon the values-square of the processing square qua the values-square of the extended processing square is the structural key for resolving why the sum of two regular bodies of Nth-dimensional space does not constitute another regular body of the Nth-dimensional space.

In the present studies I have applied the above structural key in three different ways to prove Fermat's Last Theorem. These proofs are submitted not only for the purpose of supplying a proof of this unsolved theorem, but also with the aim of appealing to other scholars to approach the main challenges within their disciplines through the Vedic wisdom.

The Vedic perspective on methodology integrates objectivity with subjective experience and as such those who are trained in objective methodology are required only to learn how to supplement this approach with their own subjective experience of the Vedic Reality. For this I feel highly privileged to be at the feet of His Holiness Shri Pada Babaji who initiated me for the Shri Pada processing line to process this quarter by quarter, and at the feet of His Holiness Maharishi Mahesh Yogi who initiated me for the Maharishi processing line to process the fourth quarter as the integration of the first three quarters. I am also highly obliged to Professor Krishnaji, Chairman, Indian Institute of Maharishi Vedic Science and Technology, for experienced guidance and personal interest in these and other studies at the Institute.

Proof by Direct Comparison

Overview

This section works out an exhaustive comparison by arranging $(Z-1)^2$ values of $S^N = X^N + Y^N$, $1 \leq X \leq Z-1$, and $1 \leq Y \leq Z-1$ as a values-square and comparing them with Z^N. These values admit groupings as as $2Z-3$ diagonals of the said square. The first Z–1 diagonals immediately give values of $S < Z$. For comparison of the remaining diagonals, the inequality $X < S < X + Y/3 < 4X/3$ is used leading to the conclusion that $S \neq Z$ for all values of the values-square.

Statement of the Theorem:

1. Fermat's Last Theorem may be stated as follows: Given a natural number $N \geq 3$, if $S^N = X^N + Y^N$, $1 \leq X \leq Z-1$ and $1 \leq Y \leq Z-1$, where X and Y are natural numbers, then $S \neq Z$.

Mode of Proof:

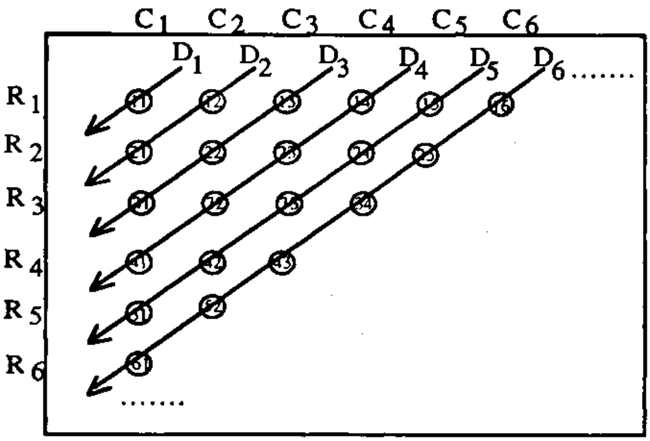
2. For the direct exhaustive comparison of the $(Z-1)^2$ values of S with Z, we may arrange these values as a values-square of (Z–1) rows and (Z–1) columns, in short to be written as V.S.Z., as follows:

Columns / rows	C_1	C_2	....	C_{Z-1}
R_1	$1^N + 1^N$	$1^N + 2^N$	....	$1^N + (Z-1)^N$
R_2	$2^N + 1^N$	$2^N + 2^N$	....	....
....	....	....	....	....
R_{Z-1}	$(Z-1)^N + 1^N$	$(Z-1)^N + 2^N$	....	$(Z-1)^N + (Z-1)^N$

3. Let $S^N = X^N + Y^N = O_{xy}$. For convenience sake, O_{xy} can be represented as xy within a circle, namely (xy) (to be referred as value point xy or simply a point). In terms of these notations, the values-square emerges as a display of value points on a format of a geometrical square as follows:

(11)	(12)	(13)	(14)	(15)	......	(1y)	......
(21)	(22)	(23)	(24)	(25)	......	(2y)	......
(31)	(32)	(33)	(34)	(35)	......	(3y)	......
....	.....	.....	.....	.....	......	.....	......
(x1)	(x2)	(x3)	(x4)	(x5)	......	(xy)	......
....	.....	.....	.....	.....	......	.....	......

4. To prove the theorem it suffices to prove that $Z^n \neq$ (xy). For this we consider the exhaustive comparison of the $(Z-1)^2$ values as diagonals D_1 to D_{2Z-3}. The diagonal D_t consists of values (xy) such that $X + Y = t + 1$. For convenient reference and handling, the diagonals may be depicted as follows:



5. The first characteristic of the values of the diagonal D_t is that $t + 1 = X + Y$ and as such $S^N = O_{xy} < (t + 1)^N$. Therefore, when $t \leq Z-1$, $S < Z$.

6. Hence, it only remains to compare the values of D_t for $t > Z$. For this we use the general inequality $X^N < S^N < (X + Y/3)^N$ for $X > Y$. The values-square V.S.Z. is suitably extended to the values-square V.S.4Z/3 and we have the compairson of the values of diagonals D_t, $Z \leq t \leq 2Z-3$.

Proof

Step 1:

7. The general inequality of natural numbers mentioned above is:

$$X^N < S^N = X^N + Y^N < (X + Y/3)^N \text{ if } X > Y, N \geq 3.$$

8. The above inequality leads to the conclusion:

$$X < S < X + Y/3$$

9. If $S = Z$, then $X < Z < X + Y/3 < X + X/3 = 4X/3$.

Step 2:

10. As the maximum value of X is Z–1, therefore

$$Z < 4\,(Z-1)/3$$

Step 3:

11. As $X < Z < X + Y/3$, therefore $Y/3 > Z-X$.

12. Now if $X = 2(Z-1)/3$, then $Y/3 > Z-2(Z-1)/3 = (3Z-2Z + 2)/3$.

13. Therefore, for $X = 2(Z-1)/3$, $Y > Z+2$ but $Y \leq Z-1$.

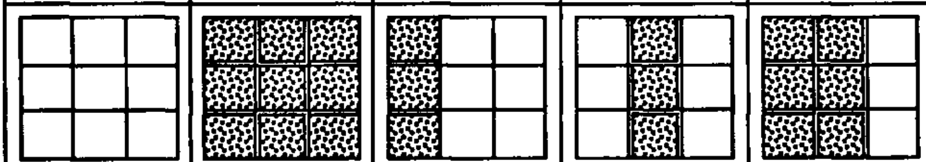
14. Hence for $X = 2(Z-1)/3$, we cannot find out any value of $Y \leq (Z-1)$, which means that for $X = 2(Z-1)/3$, $X^N + Y^N \neq Z^N$ for $1 \leq Y \leq Z-1$. Therefore, for the case of $X > Y$, the only values which need be compared to see if any of the values of the values-square is equal to Z^1 are the values for which X is greater than $2(Z-1)/3$.

Step 4:

15. Now divide the values-square V.S.Z. of Z–1 rows and Z–1 columns into three parts, columns C_1 to $C_{(Z-1/3)}$, $C_{(Z-1/3)+1}$ to $C_{2(Z-1/3)}$ and $C_{2(Z-1/3)+1}$ to $C_{(Z-1/3)}$. The first two parts, that is, C_1 to $C_{(Z-1/3)}$ and $C_{(Z-1/3)+1}$ to $C_{2(Z-1/3)}$, are values for which X is less than 2(Z–1)/3. Therefore, the values of these two parts, columns C_1 to $C_{(Z-1/3)}$ and $C_{(Z-1/3)+1}$ to $C_{2(Z-1/3)}$, are not equal to Z^N.

16. For easy comprehension and holistic view we may, with the help of the following figure, depict the above situation; the values of the shaded portion in C4 need not be compared.

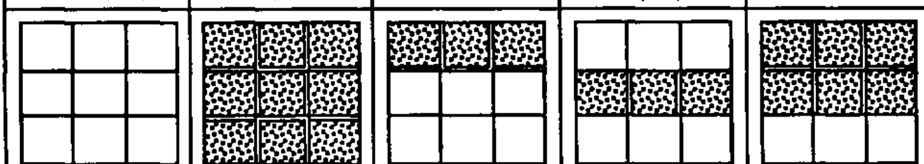
THREE PARTS OF THE (Z-1) COLUMNS				
Format	Values-square	Ist Part	IInd Part	Ist & IInd Parts
Columns (Z-1)	Columns (Z-1)	Columns (Z-1)/3	Columns (Z-1)/3	Columns 2(Z-1)/3
	C1	C2	C3	C4



Step 5:

17. Now if Y > X, then in terms of the logic of step 4, we can immediately conclude that the values of rows R_1 to $R_{2(Z-1)/3}$ are not equal to Z^N. Similarly to the values as columns, the values as rows can be geometrically depicted; the values which need not be compared are as shown in the shaded portion in R4 of the following figure.

THREE PARTS OF THE (Z-1) ROWS				
Format	Values-square	Ist Part	IInd Part	Ist & IInd Parts
Rows (Z-1)	Rows (Z-1)	Rows (Z-1)/3	Rows (Z-1)/3	Rows 2(Z-1)/3
	R1	R2	R3	R4

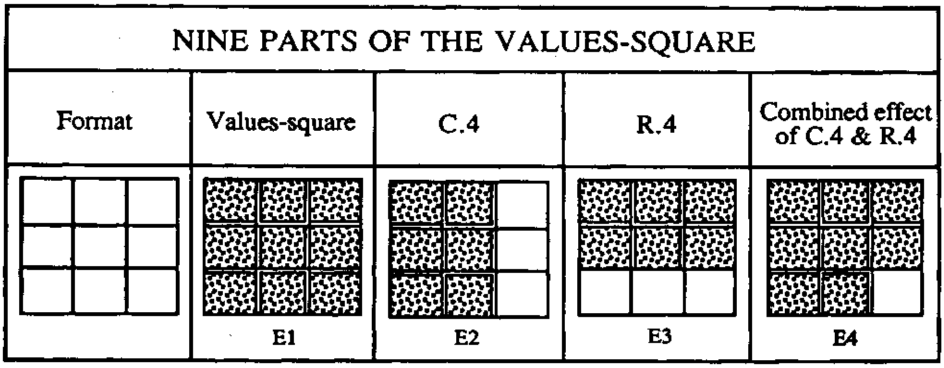


Step 6:

18. The combined effect of steps 4 and 5 is that out of the values of the values-square V.S.Z. of Z–1 rows and Z–1 columns, the values of the eight parts out of the total nine parts need not be considered, as none of them can equal Z^N. The ninth part is the values

from columns $C_{2(Z-1/3)+1}$ to $C_{(Z-1/3)}$ and rows $R_{2(Z-1/3)+1}$ to $R_{(Z-1/3)}$.

19. The combined effect is geometrically depicted as follows; the values of the shaded portion of E4 are not equal to Z^N.



Step 7:

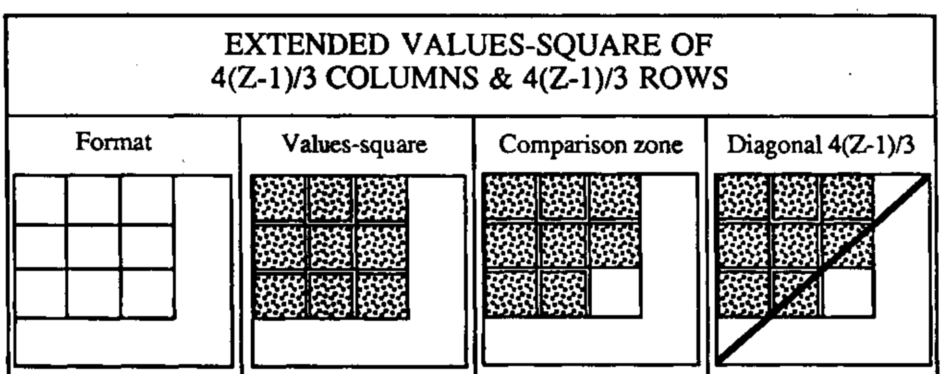
20. Call the unshaded portion of E4 the comparison zone. For comparison of the values of this portion, we resort back to the general inequality $X < S < 4X/3$. As the maximum value of X can be Z–1, then we may extend the values-square by adding a further (Z–1)/3 columns and (Z–1)/3 rows.

Step 8:

21. The values on the diagonal D_t are of the form $X^N + Y^N$ with $X + Y = t + 1$. When $t = 4(Z-1)/3$, $X + Y = 1 + 4(Z-1)/3$. For this, X would be greater than Z–1, which is not permissible. Hence, we have to consider the values only until the diagonals D_t with t less than or equal to 4(Z–1)/3.

Step 9:

22. The values of the diagonals D_t with $t \leq 4(Z-1)/3$ are contained within the shaded portion of E4. In other words, none of the values of these diagonals form part of the comparison zone.



Step 10:

23. To rule out the remote chance of the value of the diagonal D_t, where $t = 4(Z–1)/3$, as the corner of the comparison zone, being equal to Z^N, we may compare the values of this diagonal to see if any of them can be equal to Z^N.

24. The general value of the diagonal D_t, $t = 4(Z–1)/3$, is

$$S^N = [4(Z-1)/3-r]^N + [1+r]^N,\ r = 0, 1, 2, 3 \ldots .$$

25. Therefore, $4(Z–1)/3–r < S < 4(Z–1)/3–r + [1+r]/3$.

26. If $r = (Z–1)/3$, then $Z < S < Z + (1+r)/3$.

27. Therefore, $S > Z$.

28. If $r = (Z–1)/2$, then the inequality of paragraph 25 will reduce to an inequality leading to the conclusion that $S < Z$. The steps are as follows:

$4(Z-1)/3-r < S < 4(Z-1)/3 - r + [1+r]/3$

$\therefore\ 4(Z-1)/3 - (Z-1)/2 < S < 4(Z-1)/3-(Z-1)/2 + [1+r]/3$

$\therefore\ [8(Z-1) - 3(Z-1)]/6 < S < [8(Z-1) - 3(Z-1)]/6 + [1+r]/3$

$\therefore\ 5(Z-1)/6 < S < 5(Z-1)/6 + [1+r]/3$

$\therefore\ S < 5(Z-1)/6 + [1 + (Z-1)/2]/3$

$\therefore\ S < 5(Z-1)/6 + (Z+1)/6 = 5Z/6 - 5/6 + Z/6 + 1/6$

$\therefore\ S < Z-4/6 < Z$

Step 11:

29. Here it may be relevant to note that consecutive values of the diagonals are of the form $(X^N + Y^N)$ and $(X - 1)^N + (Y + 1)^N$ and hence are sequentially decreasing for $r = 0$, 1, 2, 3.... Therefore, the values of the diagonal $4(Z–1)/3$ are divided into three parts. The first part consists of values S^N for $r = 0$ to $r = (Z–1)/3$ such that $S > Z$. The second part consists of values S^N for $r \geq 2(Z–1)/3$ such that $S < Z$. The third part consists of the values for $r > (Z–1)/3$ and $< 2(Z–1)/3$.

Step 12:

30. Now S can be equal to Z only for the values in the third part, which, in fact, is the middle part of the diagonal $4(Z–1)/3$. For these values $r > (Z–1)/3$ and $< 2(Z–1)/3$. Hence, the value of the diagonal for $r = 2(Z–1)/3$ does not form part of this middle part of the diagonal. In fact, using the argument from paragraph 14, the value S^N of the diagonal for $r = 2(Z–1)/3$, as compared in above paragraph, establishes that $S < Z$. Hence, in terms of Step 8, all the values of the value-square stand compared and it is found that none of them is equal to Z^N. Hence the theorem is established. Q.E.D.

Algebraic Proof

Overview

Fermat's Last Theorem, translated into algebraic language as *"$X^N + Y^N = Z^N$ has a solution in positive integers X, Y, Z only when N = 2,"* appears to focus upon the solution in positive integers, but in fact it requires the structures of the real numbers field to rule out the solutions. The fact that the ordered completeness of the set of real numbers is equivalent to the linear geometric continuum ultimately provides the rationale for the impossibility of a solution to the above equation for $N \geq 3$.

Statement of the Theorem

1. The algebraic statement of the theorem is that *"$X^N + Y^N = Z^N$ has a solution in positive integers X, Y, Z only when N = 2."*

2. In the present proof, the concepts of the arithmetical continuum of real numbers, the geometrical continuum of a straight line, and Dedekind's cuts of rational and of real numbers are being presumed. The net outcome for the present of these concepts is that there is an ordered completeness of the field of real numbers equivalent to the linear geometric continuum, and the set of real numbers can be represented as a straight line X'OX (O as an origin point representing real number 0) such that for every rational number Q we can construct a closed interval [OQ] of rational length Q, from which an interval, say [OR] of rational length R < Q, is cut out. The remaining one-sided open interval (RQ] represents an irrational length and hence an irrational number (Dedekind's section of the set of real numbers of the closed interval [OQ] in terms of the rational number R).

Proof

3. Let us take up the proof of the theorem:

Step 1: Let, for a given Z and N, there exist X and Y such that $Z^N = X^N + Y^N$ for $N \geq 3$; X, Y, Z, N natural numbers. We have to show that this leads to a contradiction. In other words we are required to establish that for any X and Y natural numbers, if $X^N + Y^N = S^N$ then S is not a natural number.

Step 2: The general relationship $1^N = 1$ immediately gives rise to the inequality that $1^N < 1^N + 1^N < 2^N$ for $N \geq 2$. Therefore, if $R^N = 1^N + 1^N$ then $1^N < R^N = 2 < 2^N$ and, as such, $1 < R < 2$ and hence R is not a natural number.

Step 3: Now the distributive law of natural numbers implies that $M(A+B) = MA + MB$. From Step 2 above it is clear that N shall be ≥ 2, and the distributive law would add a minimum of one degree to the numbers being summed. Therefore, if the theorem would hold, the same would hold only for $N \geq 3$.

Step 4: The general inequality of natural numbers is that

If $S^N = M^N + R^N$ then $M^N < S^N < (M + R)^N$.

Now for given R and M, either $R \leq M$ or $M \leq R$.

Let us presume $R \leq M$.

Therefore, $M^N / M^N < M^N / M^N + R^N / M^N < (M/M + R/M)^N$

$\therefore\ M^N / M^N < M^N / M^N + R^N / M^N < (M/M + R/M)^N$ for $R \leq M$

$\therefore\ 1^N < 1^N + X^N < 2^N$ where $X = R/M \leq 1$ because $R \leq M$

$\therefore\ 1^N < U^N < 2^N$ where $U^N = 1^N + X^N$ and $X < 1$

Let $U^N = 1^N + (R/M)^N$

$$= \{1^N + (1/M)^N\} + \{(R/M)^N - (1/M)^N\}$$

$$= V^N + T^N$$

where $V^N = \{1^N + (1/M)^N\}$

and $T^N = \{(R/M)^N - (1/M)^N\}$.

Now let us take up V and T to ascertain if they are natural numbers or rational numbers or irrational numbers.

Case 1: Suppose V is a natural number.
From above, $V^N = \{1^N + (1/M)^N\}$
$\therefore V^N . M^N = M^N + 1^N$.
$\therefore M^N$ divides $M^N + 1^N$ by the unique factorization theorem of natural numbers. This is a contradiction. Hence V is not a natural number.

Case 2: Suppose V is a rational number P/Q such that P and Q have no common factor.
From above, $P^N M^N = Q^N (M^N + 1^N)$. But P^N does not divide Q^N, and M^N does not divide $M^N + 1^N$. Hence $M^N = Q^N$ and $P^N = M^N + 1$. This leads to a contradiction as $M^N < M^N + 1 < (M + 1)^N$,
$\therefore M^N < P^N < (M + 1)^N$
$\therefore M < P < (M + 1)$
$\therefore$ P is not a natural number while in fact P is a natural number.

Conclusion: Case 1 and 2 together lead to the conclusion that V is an irrational number.

Case 3: Suppose T is a natural number.
Then $T^N = \{(R/M)^N - (1/M)^N\}$
$\therefore T^N \cdot M^N = (R^N - 1^N)$.
$\therefore M^N$ divides $R^N - 1^N$ by the unique factorization theorem, but $M > R$, hence a contradiction.
Therefore, T is not a natural number.

Case 4: Suppose T is a rational number P/Q such that P and Q have no common factor.
From above, $P^N \cdot Q^N = Q^N (T^N - 1^N)$ but P^N does not divide Q^N and M^N does not divide $Y^N - 1^N$. Therefore, $Q^N = M^N$ and $P^N = Y^N - 1^N$.
This also leads to a contradiction,
as $P^N + 1^N = Y^N$
$\therefore P^N < Y^N < (P+1)^N$
$\therefore$ Y is not a natural number, but in fact Y is a natural number.
Therefore, T is not a rational number.

Step 5: Hence, in the equation $U^N = V^N + T^N$, V and T are irrational numbers. Therefore, we have to look for solutions of this equation in the field of real numbers. Here it may be relevant to note that U^N, V^N, and T^N are rational numbers, so $U_1 = U^N$, $V_1 = V^N$ and $T_1 = T^N$ give rise to the equation $U_1 = V_1 + T_1$ where U_1, V_1 and T_1 are rational numbers. This has the obvious solution in the field of rational numbers. However, when the solution is required for $U^N = V^N + T^N$, where V and T are irrational numbers, we have to shift to the field of real numbers.

Step 6: The field of real numbers constitutes the arithmetical continuum which is equivalent to the linear geometrical continuum of a straight line. Now the solutions of the equations $U^N = V^N + T^N$ and $M^N \cdot U^N = M^N \cdot V^N + M^N \cdot T^N$ are directly linked up. Therefore, the solution of the equation $(MU)^N = (MV)^N + (MT)^N$, which is nothing but $S^N =$

$M^N + R^N$, is to be attempted in the field of real numbers.

Step 7: As S, N, M, and R are natural numbers, therefore, $S^N = S_1$, $M^N = M_1$, and $R^N = R_1$ as well are natural numbers. The natural number P can be accepted as a rational number of the form P/1. Hence S_1, M_1, R_1 are rational numbers.

Step 8: The arithmetical continuum and linear geometrical continuum being equivalent, with respect to every real number we can construct a length on the straight line OX (where O is the origin point representing the real number 0). Now S_1 being the rational number, we can construct a closed interval [OS] on the straight line OX of rational length S_1. As $S_1 > M_1$, and as M_1 is also a rational number, therefore, we can cut a closed interval $[OM_1]$ of rational length out of the closed interval $[OS_1]$.

Step 9: Therefore, $[OS_1] - [OM_1]$ is a length of the one-sided open interval $(M_1\ S_1]$.

Step 10: This interval $(M_1\ S_1]$ is of an irrational length as the rational number M_1 **is not** part of it and all real numbers greater than M_1 and less than or equal to S_1 are in this interval. Therefore, $[OS_1] - [OM_1] = S_1 - M_1$ is irrational. Therefore, $S_1 - M_1 = R_1$ is irrational. This a contradiction as, $R_1 = R^N$ is a natural number. Therefore, our assumption that $S^N = M^N + R^N$ where S, M, N, R are natural numbers is wrong. Hence the theorem is established. Q.E.D.

Geometrical Proof

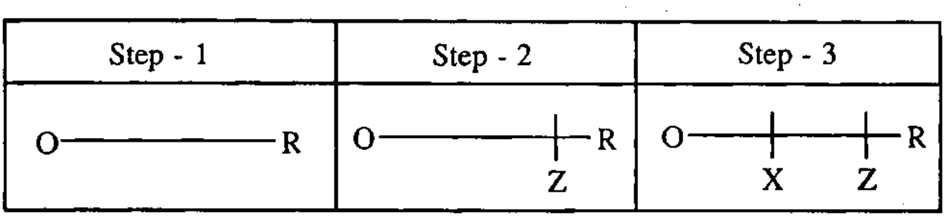
Overview

Fermat's Last Theorem is essentially a multi-dimensional regular bodies theorem. Geometrical spaces constitute an infinite geometrical continuum. An interval of rational length (closed interval) does not admit a division as the sum of two closed intervals (of rational lengths) and so $z^n \neq x^n + y^n$. Regular bodies constitute a sequence ($a^n/2na^{n-1}$, n=1, 2, 3. . .). As such 1/2 is a structural unit for the dimensional unit 1.

Introductory Remarks

1. Knowledge of the concepts of the arithmetical continuum of real numbers, the geometrical continuum of a straight line, and Dedekind's cuts of rational and real numbers is assumed.

2. Natural numbers n=1,2,3... are accepted as rational numbers of the form 1/1, 2/1, 3/1, ...



3. We may represent the real numbers on the straight line with the help of rational numbers. Proceeding in steps, begin by drawing a line OR such that O is the origin

point representing the number zero (0). As a second step, for a given rational number Z, we may cut a closed interval [OZ] of length OZ = Z from the line OR. As a third step, for any rational number X < Z, we can cut a closed interval [OX] of length OX = X from the interval [OZ] of length OZ = Z.

4. The interval [OZ] gets divided into two parts, namely closed interval [OX] and the one-sided open interval (XZ]. Both the parts, namely [OX] and (XZ], do not have any common point. All numbers rational and irrational less than or equal to X are in [OX], and all numbers rational or irrational greater than X are in (XZ]. As such, the interval (XZ] is of an irrational length.

Fermat's Last Theorem

5. Statement: "It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree."— Fermat

6. In modern mathematical language this is restated as: $x^n + y^n = z^n$ admits a solution in natural numbers only for n = 2. That means for given natural numbers z, n, and x, with x < z, we cannot find a natural number whose nth degree is equal to $z^n - x^n$.

Proof

7. Let $V = z^n - x^n$.

8. z, n, and x are natural numbers, therefore z^n and x^n are also natural numbers. Let $Z = z^n$ and $X = x^n$. Therefore, V = Z-X.

9. As Z and X are natural numbers, so the closed interval [OZ] of length Z cuts a rational length on the real line OR. Similarly, the closed interval [OX] of length X cuts a rational length from the interval [OZ].

10. V = Z-X = [OZ]-[OX] = (XZ] = irrational length.

11. Hence, $V \neq N^n$ for every natural number N, as N^n is a natural number so would equal a rational length and not an irrational length.

12. The above proves the theorem but for the restriction for the n to be 3.

Restrictions for n to be 3

Rationale for the restrictions:

13. To arrive at the rationale for the restrictions for n to be 3, we have to go back to our definition of the dimension in terms of its geometrical representation.

14. The definition of dimension cannot be expressed except within the manifested world of three-dimensional objects, thereby permitting its arrangement as a linear geometrical continuum of straight lines, each a mathematical model of the arithmetical continuum of real numbers. In concrete terms, geometry accepts dimension as that which completely permits representation as a straight line, and the geometrical universe following is the three-dimensional space with the three dimensions nothing but straight lines. In fact, here lies the rationale and the answer to the question why restrictions are placed upon n to be 3. However, comprehension of the manifested world cannot be accepted as a proof as such of the existence or otherwise of the higher- dimensional spaces. For this we have to have a purely mathematical approach.

15. For this, we may have insight into the internal structure of N^n by accepting it

to be a regular body of length N of nth-dimensional space (for the present take as a definition that degree n represents the number of dimensions).

The First Dimensional Regular Body

16. To have proper insight into the internal structure of higher-dimensional regular bodies, we would be required to pin-point the rationale as to why the very first-dimensional regular body (1^1) does not permit division into smaller regular bodies of the first-dimensional space.

17. The answer is because of the very definition of the natural number 1. The natural number 1 is the smallest natural number, so the question of the existence of still smaller natural numbers does not arise. Hence $1^1 \neq X^1 + Y^1$, where X and Y are natural numbers.

18. The above situation deserves proper mathematization. Firstly, it takes one as undefined. Simultaneously it gives us the freedom to accept any linear length, may it be equal to rational or irrational units/numbers, as a linear unit and hence "one."

19. Using the above freedom of choice to accept any linear length as the dimensional unit helps us to reduce any regular body N" to the first regular body of nth-dimensional space (Γ), where N = one unit. This also can be taken as the first regular body of the first-dimensional space (1^1) since $1^n = 1 = 1^1$.

20. Therefore, in order to understand the internal structural arrangements of dimensional regular bodies, we have no option but to consider the structural knot responsible for transforming unit linear length into a rational fraction plus an irrational fraction.

21. Designate the point of study as a **structural knot of dimensional regular bodies.** We focus on the statement that one linear unit is equal to a half-unit rational length plus a half-unit irrational length.

22. Before we take up the above statement for elaboration, I would like to add here that the rationale for the above lies in the first principle (of manifested world admitting structures) that the structural unit is half as compared to one as a dimensional unit.

Definition of a Straight Line

23. One attribute of the straight line is that it has a middle point. A second attribute of the straight line is that it has a minimum of three points. It may not be possible to give a precise definition of a straight line in terms of some attributes. However, the basic attributes of the straight line would help us settle some practical definitions. The practical definitions may have worth for any domain of practical importance but the same may not be acceptable to mathematics. A precise mathematical definition of a straight line can be provided by the arithmetical continuum of the real numbers. The format beneath the ordered display of the set of real numbers would mathematically qualify to be designated as a straight line. As such this is accepted as the definition of the straight line for the purpose of expressions of dimensional frames, as well as for the purpose of cutting lengths from a regular body.

Structural Unit Versus Dimensional Unit

24. Now let us take up the challenge of the structural knot (paragraph 21). Here the situation proceeds in two steps: firstly, in terms of the answer to the poser, if we

can transcend the three dimensional geometrical universe and have a geometrical continuum, and secondly, in terms of the fine connection between the structural unit and the dimensional unit maintaining the ratio of 1/2 : 1.

Square and Cube

25. For both the steps, initially we may proceed with the structural commonness of the square as a regular body of a two-dimensional space and the cube as a regular body of a three-dimensional space.

26. The significance of the structural commonness of a square and a cube is that the ratio of area and perimeter of the square, and volume and surface area of the cube, admit sequential order as a function of the degree of the dimensional space. We may define the degree of n-dimensional space as n. These ratios for length '**a**' of the regular bodies is:

$$a^2 : 4a \text{ and } a^3 : 6a^2$$

27. The above ratios have the formulation $a^n/2na^{n-1} : 1$ for n = 2 and 3.

28. The above sequential formulation immediately supplies the key to unlock the structural knot of the dimensional regular body. It is that for any dimensional regular body the domain part a^n (which for the square is the area of the square and for the cube is its volume) and the frame part $2na^{n-1}$ (which for the square is its perimeter and for the cube is its surface area) maintain a ratio dependent upon **1/2,** the length **(a)** and the dimensional degree **(n).** This ratio comes to be **a/2n.** The structural significance is that for any **a** and any **n,** there comes into play the factor **1/2.** This factor, which may be accepted as a halving-factor, is responsible for providing the required structural knot to bind the structural arrangements to constitute dimensional regular bodies. This is the structural key to unlock the structural knots of dimensional regular bodies of all lengths and all degrees. This factor being free of the length and degree of dimensional regular bodies acquires universal application. Hence, even the regular bodies of unit length of all the dimensional spaces of any degree also admit the factor **(1/2).** By this means we gain insight into the internal structural arrangements of 1^1, $1^2, 1^3 \ldots 1^n$.

Framed Domains Sequence

29. Before we investigate the internal structure of regular bodies of unit length of different dimensional spaces, let us understand the sequence $[a^n/2na^{n-1}$ for $n = 1, 2, 3 \ldots]$. This sequence may be defined as a framed domains sequence and its individual terms as individual framed domains or simply framed domains. The reason for the choice of terminology is that the regular bodies have their domain part contained within the frame part and thus can be designated as framed domains.

First Framed Domain

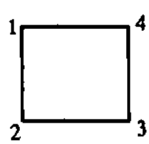
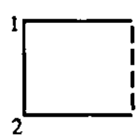
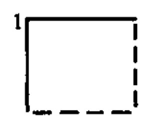
30. The first member of the framed domains sequence $[a^n/2na^{n-1}$ for $n = 1, 2, 3 \ldots]$, that is $a^n/2na^{n-1}$ for $n = 1$, that is $a^1/2 \times 1a^{1-1}$, that is **a/2,** is the first framed domain. This indicates that out of any closed linear interval we can remove one closed interval of half length while the second half would not be a closed interval. Hence the structural unit is half of the dimensional unit, which essentially is the linear unit. One may revert back to this framed domain after having been through the internal

structure of second and third framed domains, which are the well-known geometrical figures of a square and a cube, to fully realize this implication.

Internal Structure of a Square

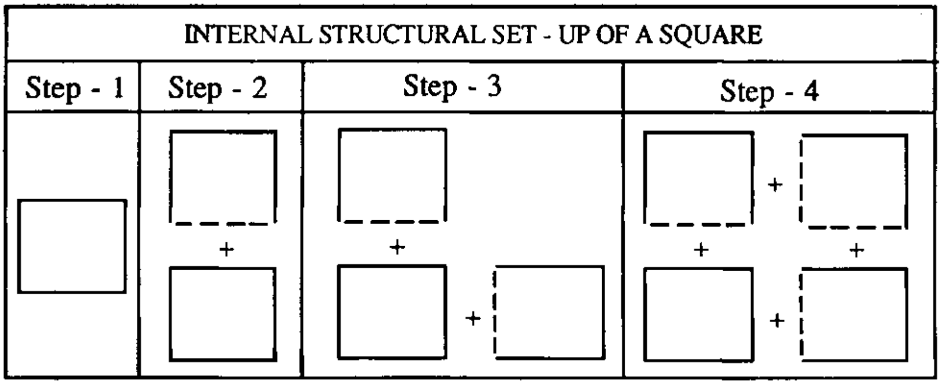
31. The square is a two-dimensional regular body with the same length on both dimensional lines. Taking length to be **a** units, the square would have an area $\mathbf{a}^2$ and a perimeter **4a.** By suitably choosing a linear unit, we can have the length of a square equal 1 (linear unit). The length 1 (linear unit) would mean a closed interval of length 1 (linear unit). Structurally this would mean that we can divide said closed interval into two intervals, one of which is a closed interval of length **1/2.**

32. Algebraically we know $(A+B)^2 = A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$. Geometrically, taking A = [OX], a closed interval, and B = [OX), a one-sided open interval (to be referred to as an open interval), the above algebraic equality would account for the internal structure of the square of length [OX].

STRUCTURAL BREAK - UP OF A SQUARE			
Area	$[OX]^2$	2[OX)[OX]	$[OX)^2$
Boundary lines	4x1 =4	2x3 =6	2x1 =2
Corner points	4x1=4	2x2=4	1x1=1
TWO DIMENSIONAL BLOCKS			

Structural Break-Up of a Square

33. Geometrically the closed interval [OO'] can be decomposed as the closed interval [OO'] = [OX] + (XO'] = [OX] + [OX), where X is the middle point of the closed interval [OO']. The structural break-up of a square of length 1 (linear unit) can be in terms of the structural break-up permissible by the linear unit as a closed interval [OX) of **1/2** unit length and open interval [OX) of **1/2** unit length. The above figure shows the structural break-up of a square of length [OO'] with X as its middle point. The algebraic equality $[OO']^2 = [OX]^2 + 2[OX][OX) + [OX)^2$ when translated into geometric equality, as has been shown in the above figure, would divide the geometrical square into four geometrical squares. Out of these four squares, the first square represented by $[OX]^2$ is a complete square with an area equal to the square of the length [OX], a perimeter equal to 4 times the length of [OX], and the four boundary lines and four corner points intact. The second and third squares represented by 2[OX][OX) are not complete squares, since one boundary line and two corner points are missing. The fourth square as well is not complete, since its two boundary lines and three corner points are missing.



34. The original square may be reconstituted in the following four steps as depicted above. As a first step, out of the four squares take the square with four boundary lines and four corner points intact. As a second step take one more square out of the remaining three squares with only one boundary line and two corner points missing. When this square is added to the square of the first step, it constitutes a rectangle of area equal to half the area of the original square.

35. Here it may be relevant to note that geometrically there remains no vacuum at all when the two squares of equal length, one of them with its one boundary line missing, are joined. This is because [OX] + [OX) = [OX] + (XO'] = [OO], thus providing a continuum throughout the boundary line along which the two squares are joined to constitute a rectangle.

36. Now as a third step, we may take one more square whose one boundary line is missing from the remaining two squares. When this square is joined with the rectangle composed in the first two steps, the missing boundary line, as is shown in the above figure, becomes a continuum in terms of the half boundary line of the inner length of the rectangle.

37. For a fourth step, join the remaining fourth square, whose two boundary lines are missing, with the geometrical figure formed as a result of the first three steps. As is evident from the above figure, one of the missing boundary lines would become a continuum in terms of the upper half of the boundary line of the inner length of the rectangle left uncovered until Step 3, while the second missing boundary line of this fourth square (as depicted as Step 4 in the above figure) would become a continuum in terms of the inner boundary line of the third square.

38. This internal structural arrangement of the square is significant in several ways. Two of these which have vital bearing for the present are that the square has nine structural points out of which eight are symmetrically located around the central ninth point where all the four squares are joined, and that when out of the square of rational length, a square of rational length is cut out, the remaining portion of the original square consists of three parts, none of which has all the four boundary lines intact.

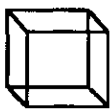
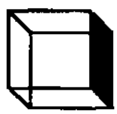
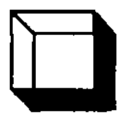
Internal Structure of a Cube

39. The cube is a three-dimensional regular body with the same length on all the three-dimensional lines. Taking the length to be **a** units, the cube has volume $\mathbf{a}^3$ and

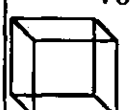
surface area $6a^2$. By suitably choosing a linear unit, we can have the length of the cube = 1 (linear unit). The length 1 (linear unit) would mean a closed interval of length 1 (linear unit). Structurally this would mean that we can divide the said closed interval into two intervals, one of which is a closed interval of length **1/2.**

40. Algebraically we know that $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$. Geometrically taking A = [OX], a closed interval, and B = [OX), a one-sided open interval (to be referred to as an open interval), the above algebraic equality would account for the internal structure of the cube of length 2[OX].

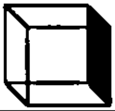
Structural Break-Up of a Cube

TOTAL AVAILABLE SURFACES 36				
Volume	$[OX]^3$	$3[OX)[OX]^2$	$3[OX)^2[OX]$	$[OX)^3$
Surfaces	6x1=6	5x3 =15	4x3 =12	3x1=3
THREE DIMENSIONAL BLOCKS				

41. Geometrically we have [OO'] = [OX] + (XO'] = [OX] + [OX), where X is the middle point of the closed interval [OO']. The structural break-up of a square of length 1 (linear unit) can be had in terms of the structural break-up permissible by the linear unit as a closed interval [OX] of **1/2** unit length and an open interval length [OX) of **1/2** unit length. The above figure shows the structural break-up of a cube of length [OO'] with X as its middle point. The algebraic equality $[OO']^3 = [OX]^3 + 3[OX]^2 [OX) + 3[OX] [OX)^2 + [OX)^3$, when translated into a geometric equality, as has been shown in the above figure, divides the geometrical cube into eight geometrical cubes.

Length	Volume	Surfaces	Edges	Corner - points
[OX] [OX] [OX]	 $= [OX]^3$	6	12	8

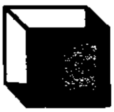
42. Out of the above eight cubes, the first cube, represented by $[OX]^3$, is a complete cube having a volume equal to a cube of the length [OX], surface area equal to six times the area $[OX]^2$, and all the six surfaces and eight corner points intact.

Length [OX) Breadth [OX] Height [OX]	Volume  =[OX) [OX]^2	Surfaces 5	Edges 8	Corner - points 4

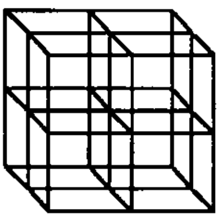
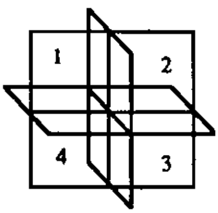
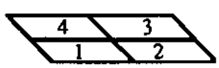
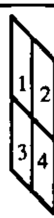
43. The second, third, and fourth cubes represented by 3[OX]^2 [OX) are not complete cubes, since one surface and four corner points are missing.

Length [OX) Breadth [OX) Height [OX]	Volume  =[OX)^2 [OX]	Surfaces 4	Edges 5	Corner - points 2

44. The fifth, sixth, and seventh cubes as well are not complete cubes, since two surface lines and six corner points are missing.

Length [OX) Breadth [OX) Height [OX)	Volume  =[OX)^3	Surfaces 3	Edges 3	Corner - points 1

45. The eighth cube too is not complete, since its three surfaces and seven corner points are missing.

EXTERNAL SURFACES	INTERNAL SURFACES		
4x6 = 24	Front 4	Horizontal 4	Vertical 4
			

46. The division of the cube into eight cubes of above description can be used in reverse to reconstitute the original cube in terms of the said eight cubes. This is similar to the case of the square traced above in paragraphs 34 to 37. The said eight cubes

have in all 36 surfaces. When these are combined as above, twelve surfaces are used to form a continuum with the missing surfaces of said cubes. The remaining 24 surfaces account for the surface area of the re-composed original cube as is evident from the above figure.

Summary

47. Before we take up the general Nth term of the framed domains sequence, I would like to summarize the position emerging from the above analysis of the first three framed domains. These three cases involve common geometrical figures, namely a closed interval of a straight line, a square, and a cube of rational lengths.

48. With respect to the first framed domain, it requires a definition of a structural frame for a closed interval. With respect to the square and the cube it is obvious that the area of the square is contained within the perimeter of the square and the volume of the cube is similarly contained within the surface area of the cube. As such the perimeter and surface area respectively are the framed parts of the square and the cube. Structurally the length of the closed interval requires a middle point and as such we may define a point as a zero space figure responsible for providing a frame for the (linear) length by having placement for the frame point anywhere in between the interval. (To be specific, the placement for the frame point is not at the beginning or the end point of the closed interval.) That way only one point (to be called the middle point since it falls midway between the two end points) constitutes the frame of the first framed domain.

49. From the above, we may conclude (and tabulate as the figure below indicates) that when from the first framed domain, that is, a closed interval of a straight line, a closed interval of smaller rational length is cut out, the remaining portion constitutes an irrational length as it is missing one end point. When from the second framed domain, that is, a square of rational length, a square of smaller rational length is cut out, the remaining portion constitutes three squares out of which two squares are missing one boundary line, and one square is missing two boundary lines. Similarly, when from the third framed domain, that is, a cube of rational length, a cube of smaller rational length is cut out, the remaining portion constitutes seven cubes out of which three cubes are missing one surface, another three cubes are missing two surfaces, and the last, that is, the seventh cube, is missing three surfaces.

50. The following table evidently makes it clear that when the rational length of the regular body of the Nth-dimension (Nth framed domain) is divided into rational part (Q) and the remaining irrational part (R), the regular body gets divided into 2^N N-dimensional bodies of the form $Q^n R^{N-n}$, where n = N, N-1, N-2, ... , 2, 1, 0. These N-dimensional bodies, numbering N+1, may be called respectively, first, second, ... (N+1)th-dimensional blocks of degree N. There is only one N-dimensional body of the first-dimensional block. There are N N-dimensional bodies of the second-dimensional block. The R-dimensional block has (N.N-1.N-2... N-R/1.2.3...R) N-dimensional bodies. The last block has one N-dimensional body.

51. As a net result, the following internal arrangements of regular bodies justify our logic and conclusion of paragraphs 7 to 12 that when from the rational length, a rational portion is cut out, it leaves behind an irrational length.

SPLIT-UP OF THE Nth DIMENSIONAL REGULAR BODY OF RATIONAL LENGTH a = [OO'] INTO (N+1) DIMENSIONAL BLOCKS OF Nth DIMENSIONAL BODIES OF THE FORM $Q^n R^{N-n}$

$a^N = (Q+R)^N$

Sr. No.	Dimensional space	Framed domain	Domain / Content part	Frame part	Total Dimensional blocks on split-up as $(Q+R)^N$	Dimensional bodies of (S+1)th block $\frac{N.N-1...N-S}{1.2...S}$	Frame components missing from Sth block
1	First	$a^1 / 2a^0$	a^1	$2a^0$	1+1 = 2	1, 1	0, 1
2	Second	$a^2 / 4a^1$	a^2	$4a^1$	2+1 = 3	1, 2, 1	0, 1, 2
3	Third	$a^3 / 6a^2$	a^3	$6a^2$	3+1 = 4	1, 3, 3, 1	0, 1, 2, 3
4	Fourth	$a^4 / 8a^3$	a^4	$8a^3$	4+1 = 5	1, 4, 6, 4, 1	0, 1, 2, 3, 4
....	....	....	....	....	....	....	....
s	sth	$\frac{a^s}{2sa^{s-1}}$	a^s	$2sa^{s-1}$	s+1	$\frac{N.N-1...N-S}{1.2...S}$	0, 1, ...s

Degree of Freedom

52. Now we may take up the question why restrictions are necessary for n to be 3. In other words, the question is: why for the first and second degree of natural numbers, that is, for n = 1 and 2, the equation $Z^n = X^n + Y^n$ has solutions. Here it may be relevant to note that the division into two parts of like degrees is not possible for all Z^n, n = 1, 2. When Z and n both are equal to one, the division for $Z^n = 1^1$ is not possible, though for all Z 2 and n = 1, the desired division into two parts obviously holds by the very definition of the natural numbers. For example, consider 1, 1+1, 1+2, 1+3 ...,1+s,....

53. For n = 2, every Z^n is not decomposable as $X^n + Y^n$. Illustrations of the point are 1^2, 2^2, 3^2, 4^2. There are infinitely many values of Z for which the desired decomposition is not available. On the contrary, there are infinitely many values of Z for which the desired decomposition holds. One class of such values is Z = 5. The decomposition for this value is $5^2 = 3^2 + 4^2$. Now for any value of the form 5Z, a similar relationship also holds, namely $(5Z)^2 = (3Z)^2 + (4Z)^2$.

54. So for n = 1 and 2, the desired decomposition is not universally permissible though the same holds for infinitely many values of Z. This restricted permissibility of the decompositions for n = 1 holds for all values except Z = 1. And for n = 2 for a large number (a countable number of values of Z) the decomposition holds and for an equally large number (a countable number of values of Z), the same does not hold.

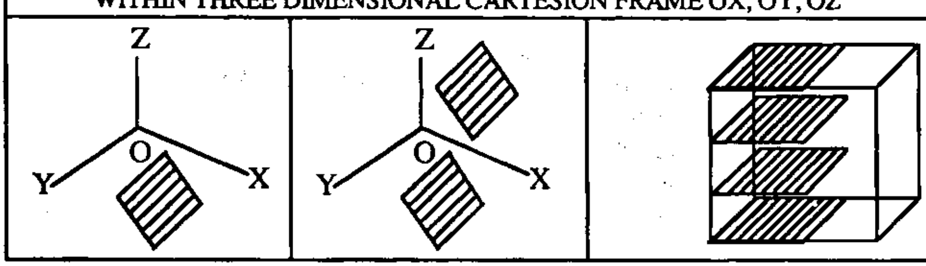
55. The reason for the restricted permissibility for n = 1 is that the absolute value of natural number 1 and the linear measure value of unit length are two distinct concepts. The natural number 1 is the absolute value in the sense that it is not dependent upon any

dimensional structures while the linear measure value of unit length is completely dependent upon the structural arrangement of a one-dimensional space, that is a straight line. The linear unit would represent a closed interval of unit length. The internal structural restrictions of a closed interval make it impossible for its division into two closed intervals. As such the natural number 1, as the first degree value of one, does not admit the desired decomposition. However, the straight line and hence a closed interval, being a first-dimensional body (figure) has two degrees of freedom within a three-dimensional space. As such, externally the linear unit is free to combine itself with a similar unit to constitute a one-dimensional body (figure) of two units. The role of the origin point of the three-dimensional frame is crucial since from it emanates dimensional lines and for each of the three-dimensional lines it remains a zero value starting point. Because of this, it is possible to have linear units simultaneously sprouting from the origin point along the dimensional lines. Hence, there are two distinct planes available for every dimensional line, which may be used by the given linear unit of any of the three dimensional lines. This two-fold freedom for the straight line accounts for the desired decomposition for all natural numbers except $Z - 1$.

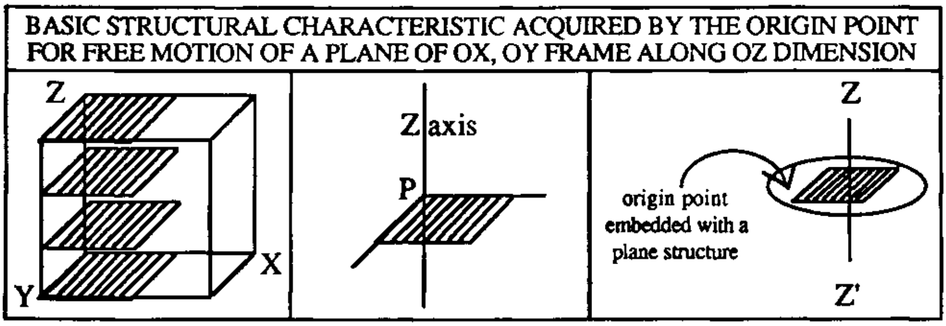
56. Turning to the plane, a square, a two-dimensional regular body, has as well, one degree of freedom in a three-dimensional space. It is because of the restricted freedom of the plane that the decomposition is not universally permissible in the case of two-dimensional regular bodies. Rather, in their case, within any finite range of natural numbers, say $1 \leq Z \leq N$, the desired decomposition generally would not be permissible and in a comparatively very small number of values of Z only the desired decomposition would hold. This is so also within a three-dimensional space, two dimensions stand restricted because of the structural format of a plane and it is left with only one degree of freedom. Here we may compare the situation with the fate of a one-dimensional body, that is, a straight line whose structural format has only one dimension. It is left with two degrees of freedom in terms of unrestricted two dimensions of the three-dimensional space.

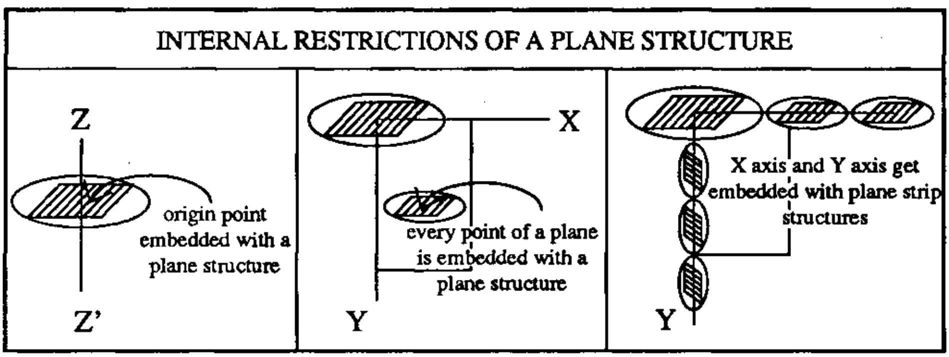
57. Now when we come to a three-dimensional body, its structural format restricts all the three dimensional lines and hence we are left with no degree of freedom. It is because of this that it is not possible to duplicate the cube. In the case of a plane, it is possible to duplicate it as a plane having one degree of freedom in a three-dimensional space. As such we can move (or pile) a plane (or identical regular bodies of a two-dimensional space, say square) along the third dimension as is evident from the following figure:

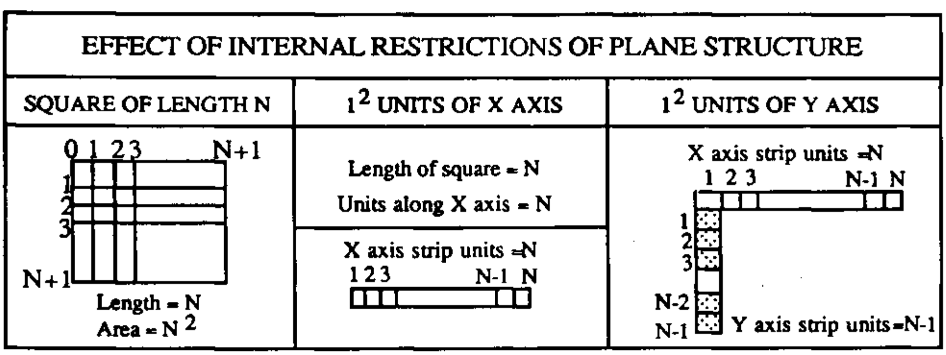
MOTION OF A PLANE HAVING ONE DEGREE OF FREEDOM ALONG OZ AXIS WITHIN THREE DIMENSIONAL CARTESION FRAME OX, OY, OZ



58. The first basic structural characteristic acquired by the origin point because of the possibility for the free motion of a plane of OX, OY frame along OZ dimensional line is that the origin point gets embedded with a plane structure. Because of this, it hardly matters which point of the Z axis is chosen as the origin point. Though the plane structure acquired by the origin point of the dimensional frame does not affect the external freedom for motion of the plane towards the OZ axis, the internal structural arrangement of the plane figures is governed by the two-dimensional format. The glaring effect of the two-dimensional format for the plane figures is that its every point acquires a two-dimensional structure. Because of this even the dimensional lines, as boundary lines of plane figures, get inseparably merged with the plane figure. As such, structurally they too can be dealt with only as plane strips as shown in the figures below.

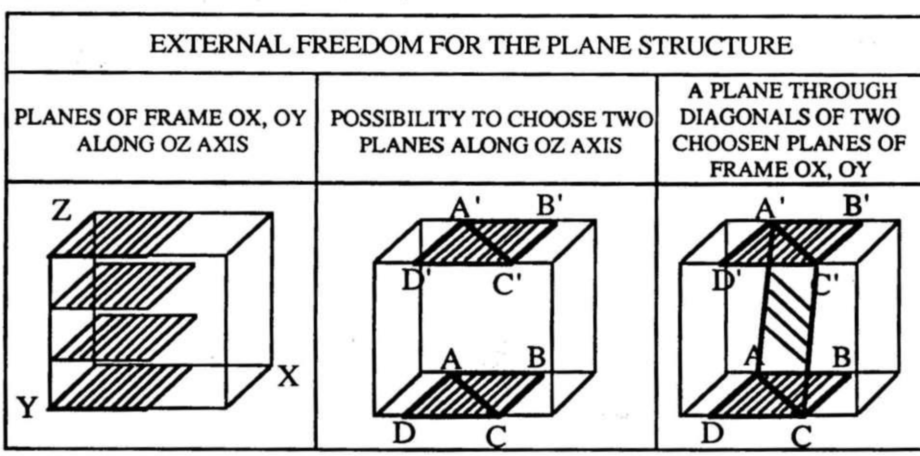






59. The immediate effect is that when we divide a square N^2, or as a matter of fact even a rectangle (NxM), of length N, then l^2 units of the X axis, as has been shown above, would be N, but the l^2 units of Y axis would remain only N-l (in the case of a rectangle, only M-l). Therefore, the re-assembled position for $l^2 = 1$ for the X and Y axis would give N x (N-l) (and in case of a rectangle N x [M-l]) which is less than N^2 (in the case of a rectangle N x (M-l) < N x M). This is the reason why in a large number of cases it is not possible to decompose Z^2 as $X^2 + Y^2$ for natural numbers Z, X, and Y.

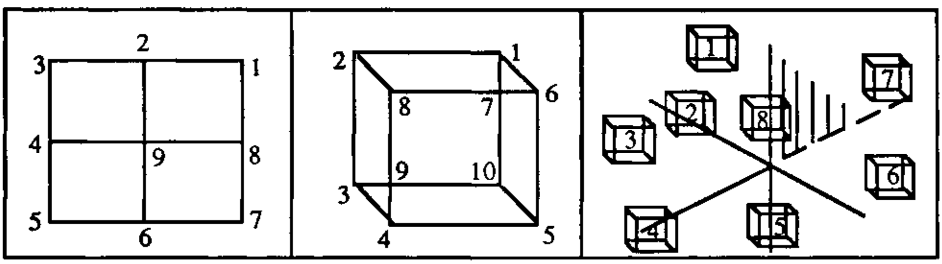
60. Now using one degree of freedom, which may be viewed as the external freedom for the plane figures, the restrictions of the two-dimensional format of the plane figures can be kept in abeyance. Here it may be relevant to note that the proof of the Pythagoras Theorem, with the help of a right-triangle, may appear as if we are using only one plane. As a matter of fact, we are using more than one plane by considering the sides of a triangle. Suppose the square of the hypotenuse of a right-angle triangle is split into two squares which equal the squares of the perpendicular and base of the said triangle or, as a converse, the reconstruction of a square of the hypotenuse in terms of the squares of the base and perpendicular. Then we have to use the internal structural arrangments permitted by the square as a regular body of the two-dimensional space. Further, we have to use eight symmetries of the square and one degree of freedom (which the square as a two-dimensional body would have) within a three-dimensional Cartesian frame.



61. As is shown above, on the two-dimensional format of OX, OY dimensional lines we can have plane figure ABCD (which may be a square or a rectangle). This figure would have a degree of freedom of motion along the OZ axis. This provides us with the possibility to choose two identical figures, say ABCD and A'B'C'D' as shown above. The diagonal, being on the one-dimensional format, would have length equal to any real number, which includes the natural numbers. The two-dimensional figure AC C'A' may be a square on a proper choice of vertices.

62. It is because of the above freedom of construction for the plane figures in a three-dimensional frame that it becomes possible to divide the square of the hypotenuse (AC

C'A') in terms of the squares on the dimensional lines of the format of the plane figure ABCD and vice-versa. Though this assignment as such is not taken up here, it may not be out of context to note that the unit square would be the basic constituent which would be used for a division process of one square into two squares or the reverse process of composition of two squares into one square. The unit square (and as such any square) has eight symmetries, and the three-dimensional regular body as well has eight corner points, and the three-dimensional Cartesian frame cuts the space into eight octants, as shown below. It is because of this that the division process and the reverse composition process is possible.

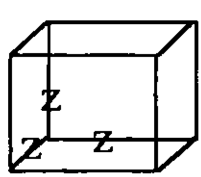
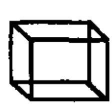


63. Hence we have justified the restriction of Fermat's Last Theorem to n ^ 3.

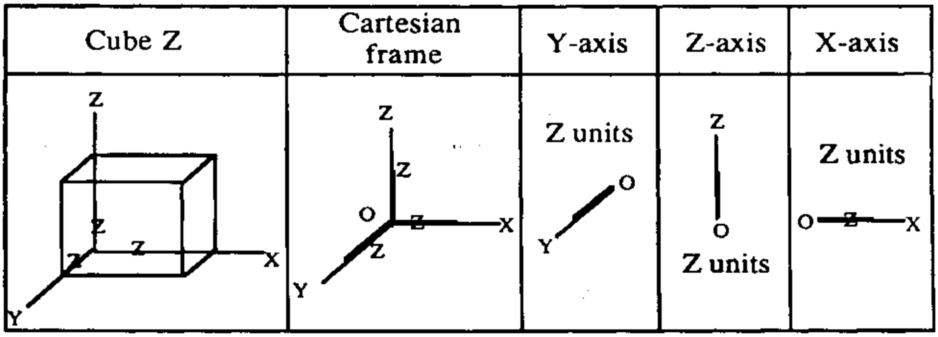
64. Now we may take up the thread of the above logic of internal restrictions of the format and the external freedom, if any is available, for the dimensional regular bodies moving from two-dimensional bodies to three- and higher-dimensional bodies.

Case of a Three-Dimensional Regular Body

65. Step 1: Let volume $V = Z^3$ cubic units. In other words, the cube Z is constituted by Z^3 unit cubes.

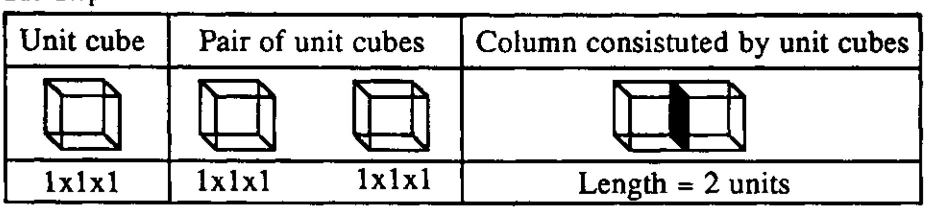
Unit cube	Cube Z	Volume of cube Z
		$= Z^3$ (1x1x1) where Z^3 is a natural number 1x1x1 is a volume unit

66. Step 2: The cube Z has Z units along the OX dimensional line as well as Z units along both of the remaining two dimensional lines.

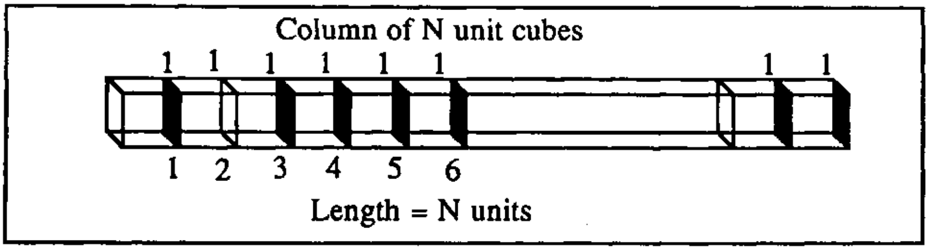


67. Step 3: Z unit cubes are required to constitute a length of Z units along the X axis.

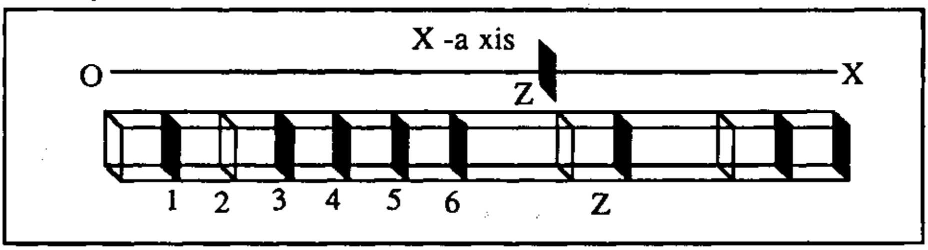
Sub-Step 1



Sub-Step 2



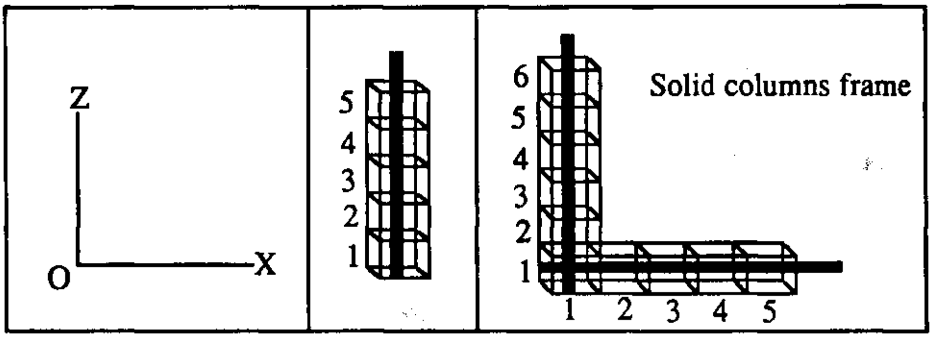
Sub-Step 3



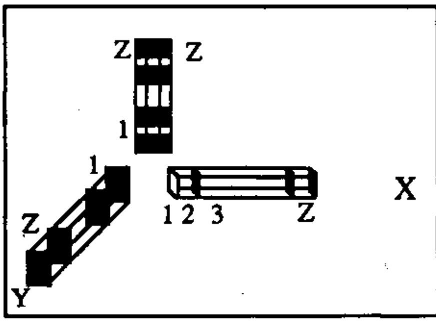
68. Step 4: Z unit cubes are required to constitute a length of Z units along the Y axis and similarly, Z unit cubes are required to constitute a length of Z units along the Z axis.

69. Step 5: The origin point of the Cartesian frame, being a dimensionless point, when the X axis and the Y axis emerge from the origin point, the dimensional length along the X axis is not affected by the dimensional length along the Y axis and vice versa.

70. Step 6: When two columns of unit cubes are joined together like the X axis and the Z axis, then the length along the Z axis is increased by one unit.



71. Step 7: When three columns of unit cubes are joined together like the X axis, Y axis, and Z axis, then the lengths along the Y axis and the Z axis are increased by one unit.



X-axis length	Y-axis length	Z-axis length
Z-units	Z+1 units	Z+1 units

72. Step 8: We had started with cube Z of length Z and volume $V = Z^3$, but have ended with a cube of volume $Z(Z+1)(Z+1)$ which is greater than Z^3, hence there is a contradiction.

Conclusion

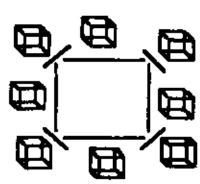
73. Therefore, the above contradiction proves that our assumption regarding permissibility of the volume of the cube to be separated and handled $1\times1\times1 = 1$ as a natural unit is wrong. Hence X^3 cubic units cannot be treated as linear units and as such the addition operation of natural numbers is not a meaningful operation. If cubic units $1\times1\times1$ are treated as linear units 1, then it would result in a contradiction. Hence, the theorem for the cubes that it is impossible to separate a cube into two cubes.

74. The above logic would be equally applicable to the bi-quadratic and higher powers as well, as the natural numbers addition operation is geometrically linear in nature; the same is meaningless and it is not applicable to the higher-dimensional units 1×1, $1\times1\times1$, $1\times1\times1\times1$....

75. Z^2 (1x1) and Z^2 (1) are two distinct units. Similarly Z^3 (1x1x1) and Z^3 (1) as well as Z^4 (1x1x1x1) and Z^4 (1) and so on, are pairs of distinct units. The addition operation which binds 1+1 is meaningless for the units 1x1 or 1x1x1, or 1x1x1x1 and so on, since (1x1) + (1x1) means an operation in terms of which two square units are to be combined, while 1+1 = 2 is the natural numbers addition operation. Unless and until we know how the square unit is transformable into a linear unit, the linear units addition operation would not be applicable. The natural numbers multiplication operation, which is intimately connected with the addition operation, remains dependent upon the addition operation only within a linear space. The moment the space stands changed from one-dimensional (linear) space to two and higher-dimensional space, the independent characteristics of the multiplication operation are displayed. As such, unless and until 1x1 is suitably defined, the addition operation will remain only the operation of one-dimensional space.

Geometrical Continuum: Fourth-Dimensional Space

76. Now we may take up the fourth number of the framed domains sequence **[$a^n/2na^{n-1}$** for **n = 1, 2, 3 ...**]. For the present it may be taken by way of definition that **[$a^4/8a^3$]** is the formulation for the regular body of fourth-dimensional space of dimensional length **a**, content part (domain part) as **a^4**, and the frame part as **$8a^3$**. In the context we may refer to a cube of dimensional length **a**, content (domain part) as volume **a^3**, and frame part as surface area **$6a^2$** as the three-dimensional regular body. Further, by way of definition, we may take the following figure as a geometrical presentation of the fourth-dimensional regular body.

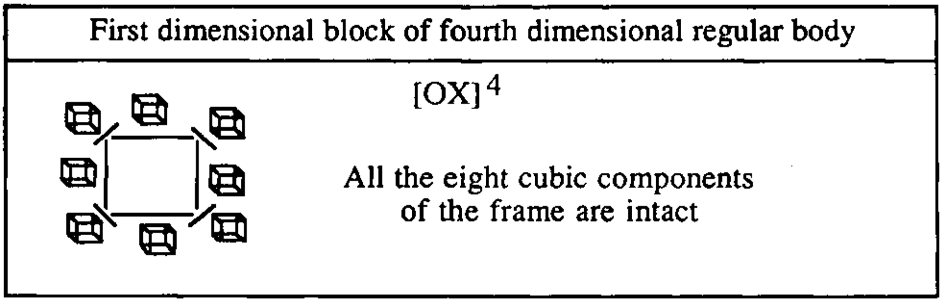
Fourth member of the framed domain sequence	
Regular body of fourth dimensional domain $a^4/8a3$	

Structural Arrangement of Fourth Framed Domain

77. Algebraically we know that $(A+B)^4 = A^4 + 6A^3B + 4A^2B^2 + 6AB^3 + B^4$. Geometrically, taking A = [OX] and B = [OX) as a one-sided open interval (to be referred to as an open interval), in terms of the above expansion, A+B = [OO'] = [OX] + (XO'] = [OX] + [OX), when X is a rational number and is the middle point of [OO']. If [OX] = X, then [OO'] = 2X.

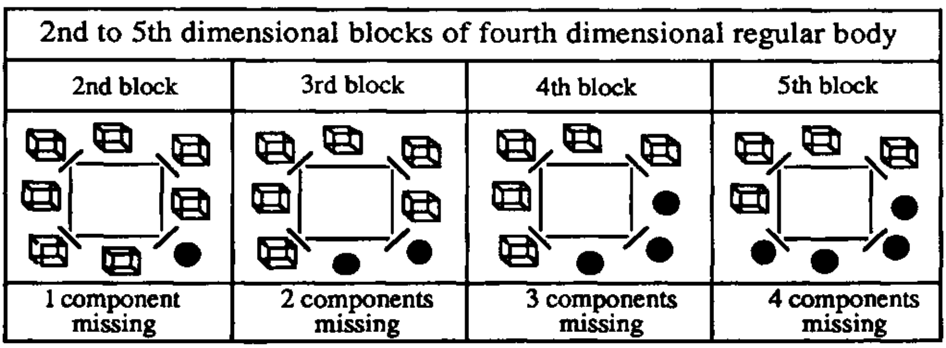
78. The algebraic equality $[OO']^4 = [OX]^4 + 6[OX]^3\ [OX) + 4[OX]^2\ [OX)^2 + 6[OX]\ \ [OX)^3$ works out the structural arrangement of the fourth framed domain. This, when translated into geometry, shall divide the fourth-dimensional body into seventeen fourth-dimensional bodies.

79. Out of the seventeen fourth-dimensional bodies, the first, namely $[OX]^4$, is a complete fourth-dimensional body in the sense that all the eight cubic components of the frame are intact.



80. We may define a body of such frame with all the eight cubic components intact as a first-dimensional block of the regular body.

81. As is evident from the above algebraic equality, in all we shall have five types of dimensional blocks. The second, third, fourth, and fifth-dimensional blocks would be structurally the fourth-dimensional bodies such that their frames are respectively missing 1, 2, 3, and 4 cubic components.



82. Therefore, the internal structural arrangement of the fourth-dimensional regular body admits a break-up as five-dimensional blocks such that these blocks have respectively 1, 6, 4, 6, and 1 fourth-dimensional bodies. Evidently, all the five-dimensional blocks are structurally distinct.

83. Now the general binomial expansion of $(A+B)^N$ would help us conclude that we shall have (N+l)-dimensional blocks. In this format, 2^N N-dimensional bodies would constitute the regular body $(A+B)^N$. As for the N-dimensional space, the format of the dimensional blocks is to remain the same, so as to structure a regular body, exactly 2^N N-dimensional bodies would be required.

84. Therefore, for N 3 and Z 3 (for Z = 1 and 2 the theorem is obvious), let $Z^N = X^N + Y^N$ where X, Y, Z, and N are natural numbers. Further, let Z > X > Y (for X = Y the theorem has been proved above, and X and Y being general, it remains to prove the theorem when one of X or Y is greater than the other). Let Z = X + R. Therefore, $X^N + Y^N = (X+R)^N$. The right side on expansion yields $X^N + NRX^{N-1} + \ldots$. Therefore, $X^N + Y^N = X^N + NRX^{N-1} + \ldots + R^N$.

Therefore, $Y^N = NRX^{N-1} + \ldots + R^N$.

Now the right side is an expression of 2^N-1 N-dimensional blocks.

The left side can have divisions into N-dimensional blocks as $Y = A+B$.

$Y^N = (A+B)^N$ yields 2^N N-dimensional blocks, while the N-dimensional blocks on the right side are only 2^N-1. Hence, the left side cannot be equal to the the right side. With this the geometrical proof of the theorem stands completed.

85. Now if in the light of the above, we try to find the significance of the internal structural arrangement of the very first regular body 1^1, where 1 is equal to an irrational length, then it really would be enchanting to see how Nature works out 1 = 1/2 (half of a rational unit) + 1/2 (half of an irrational unit). It provides us with a geometrical continuum which, as a linear continuum, is equivalent to the field of real numbers whose subfield is the field of rational numbers.

86. Obviously, the implications are many and far reaching, particularly when they demolish mental blocks of the physical world and help transcend to four and higher-dimensional spaces as a geometrical continuum of spaces whose regular bodies constitute a framed domains sequence ($a^n/2na^{n-1}$, n = 1, 2, 3 ...).